1. Hilary was examining the differences between perfect squares of numbers separated by 5. She made the following conjecture: The differences always have the digit 5 in the ones place. For example: $17^2 - 12^2 = 289 - 144 = 145$
   a) Gather evidence to support Hilary's conjecture.
   b) Is her conjecture reasonable? Explain.

2. Denyse works part time at a grocery store. She notices that the store is very busy when she works in the afternoon from 4 to 7 p.m., but it is less busy when she works in the evening from 7 to 10 p.m. What conjecture can you make for this situation? Justify your conjecture.

3. Heather claimed that the sum of two multiples of 4 is a multiple of 8. Is Heather's conjecture reasonable? Explain. If it is not reasonable, find a counterexample.

4. Prove that the sum of two consecutive perfect squares is always an odd number.

5. Prove that the following number trick will always result in 6:
   - Choose any number.
   - Add 3.
   - Multiply by 2.
   - Add 6.
   - Divide by 2.
   - Subtract your starting number.

6. Judd presented the following argument:
   Inuvik, Northwest Territories, is above the Arctic Circle, which is at a latitude of 66° north of the equator. Places north of the Arctic Circle have cold, snowy winters. Winnipeg is at a latitude of 52° north of the equator. Therefore, Winnipeg does not have cold, snowy winters.
   Is Judd’s argument reasonable? If not, identify the errors in his reasoning.

7. Is this proof valid? Explain.

<table>
<thead>
<tr>
<th>Let any number, $a$, equal $b$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a^2 = ab$</td>
</tr>
<tr>
<td>$a^2 - b^2 = ab - b^2$</td>
</tr>
<tr>
<td>$(a + b)(a - b) = b(a - b)$</td>
</tr>
<tr>
<td>$a + b = b$</td>
</tr>
<tr>
<td>$a + b - b = b - b$</td>
</tr>
<tr>
<td>$a = 0$</td>
</tr>
</tbody>
</table>

| Multiply both sides by $a$.       |
| Subtract $b^2$ from both sides.  |
| Factor both sides.               |
| Divide by $(a - b)$.             |
| Subtract $b$ from both sides.    |
| Any number equals zero.          |
CHAPTER 2 TEST

1. Determine whether each pair of angle measures can be used to prove that \( AB \parallel CD \). State how you know.
   a) \( \angle PQA = 57^\circ \) and \( \angle DRS = 57^\circ \)
   b) \( \angle AQR = 123^\circ \) and \( \angle QRD = 123^\circ \)
   c) \( \angle QRC = 57^\circ \) and \( \angle DRS = 57^\circ \)
   d) \( \angle BQR = 57^\circ \) and \( \angle QRD = 123^\circ \)
   e) \( \angle PQA = 57^\circ \) and \( \angle QRC = 57^\circ \)

2. Prove that \( AB \parallel CD \).

3. Prove that \( AC \parallel GI \).

4. a) Construct a parallelogram using a ruler and a compass.
   b) Draw diagonals in your parallelogram. Verify that your construction is accurate by measuring pairs of alternate interior angles.

5. Given: \( \angle SQT = 83^\circ \)
   \( \angle OTU = 151^\circ \)
   \( RP \parallel SU \)
   Determine the measures of all the other angles. State your reasoning.

6. Determine the measure of each interior angle of a regular 11-sided polygon.

7. Prove that the opposite sides of a regular hexagon are parallel.
CHAPTER 3 TEST

1. Determine the indicated side lengths or angle measure in each triangle.
   a) [Diagram of triangle ABC with sides labeled: AB = 72.4 cm, BC = 32°, and AC = 72°]
   b) [Diagram of triangle DEF with sides labeled: DE = 7.2 m, DF = 9.2 m, and EF = 6.3 m]

2. Solve each triangle, using the given information.
   a) \( \triangle ABC \), \( \angle A = 88° \), \( a = 15 \) cm, and \( c = 8 \) cm
   b) \( \triangle DEF \), \( \angle F = 72° \), \( d = 8.0 \) cm, and \( e = 6.0 \) cm

3. The lengths of the sides in \( \triangle RST \) are 6.0 cm, 12.0 cm, and 15.0 cm.
   Determine the measure of the smallest angle in the triangle to the nearest degree.

4. A canoeist paddles 3.8 km in a N63°E direction and then 4.2 km in a N35°W direction. Determine the length of a direct path to the canoeist's final position to the nearest tenth of a kilometre.

5. From Jennifer's position on the finish line of a rally course, she can see the flags that mark the final two check-in points. One flag is 720 m directly in front of her, and the other flag is at an angle of 70°. If the distance between the flags is 800 m, how far is the second flag from the finish line?

6. Kenny used a graphic design program to create a logo for his new business. The logo contains a triangle with side lengths of 30 cm, 35 cm, and 24 cm. Determine the angle measures and the positions of the angles in relation to the sides of the triangle.

7. Élise is planning a nature hike in a forest. Use the sketch to calculate the total distance, to the nearest hundredth of a kilometre, of the hike from \( J \) to \( K \), \( K \) to \( L \), \( L \) to \( M \), and back to \( J \).
CHAPTER 4 TEST

1. Solve for $\theta$, if $0 \leq \theta < 180^\circ$.
   
a) $\tan \theta = \frac{3}{4}$  
b) $\cos \theta = -0.8520$  
c) $\sin \theta = 0.7352$

2. Determine the indicated side length or angle measure in each triangle.
   
a) 
   \[ \begin{array}{c}
   \triangle ABC \\
   \hfill 3.0 \text{ m} \\
   \hfill 2.5 \text{ m} \\
   \hfill 4.8 \text{ m} \\
   \end{array} \]

   b) 
   \[ \begin{array}{c}
   \triangle ABC \\
   \hfill 25.6 \text{ mm} \\
   \hfill 20.4 \text{ mm} \\
   \hfill 42^\circ \\
   \end{array} \]

3. Nathan is trying to prove the cosine law, as shown below, but he has made an error in one step. Find Nathan’s error, and complete the proof.

   I drew the height outside the triangle and created two different right triangles with the same height, $h$.

   \[ \triangle ABD \quad \triangle CBD \]

   $h^2 = c^2 + (b + x)^2$  
   $h^2 = a^2 + x^2$

   $c^2 + (b + x)^2 = a^2 + x^2$
   $c^2 = -(b + x)^2 + a^2 + x^2$
   $c^2 = -b^2 - 2bx - x^2 + a^2 + x^2$
   $c^2 = -a^2 - b^2 - 2bx$

   $\cos(180^\circ - \angle ACB) = \frac{x}{a}$
   $a \cos(180^\circ - \angle ACB) = x$

   $c^2 = -a^2 - b^2 - 2b[a \cos(180^\circ - \angle ACB)]$
   $c^2 = -a^2 - b^2 + 2ab \cos \angle ACB$

   I used the Pythagorean theorem to write an equation for the height squared in each triangle.
   I made these equations equal to each other and solved for $c^2$.

   I used the smaller external right triangle to write an expression for $x$.
   I substituted this expression for $x$ into the $c^2$ equation.
   I knew that the cosine ratios for supplementary angles are opposites, so I replaced $\cos(180^\circ - \angle ACB)$ with $-\cos \angle ACB$. 

228 I Foundations of Mathematics 11: Chapter 4: Oblique Triangle Trigonometry
4. Given each SSA situation for \( \triangle ABC \), determine whether there are zero, one, or two possible triangles. Explain your reasoning.
   a) \( a = 15.5 \text{ m}, \ b = 12.0 \text{ m}, \ \angle B = 35^\circ \)
   b) \( a = 15.5 \text{ m}, \ b = 8.5 \text{ m}, \ \angle B = 35^\circ \)
   c) \( a = 15.5 \text{ m}, \ b = 17.6 \text{ m}, \ \angle B = 35^\circ \)

5. Sandra is building a ramp for biking on her street. She has already completed the 10 ft part for the incline of the ramp. She would like this part of the ramp to rise at a 15° angle and then join the second part of the ramp, which will go down to the ground. She wants the total length of the ramp to be only 13 ft horizontally.
   a) Determine the length of the second part of the ramp, to the nearest tenth of a foot.
   b) What is the angle of inclination for the second part of the ramp, to the nearest degree?

6. A carpenter measures three sides of a triangular deck, which needs painting. The side lengths are 15 ft, 14 ft, and 26 ft. Determine the area of the deck, to the nearest square foot.

7. A computer malfunction onboard a ship causes it to veer 5° off course for 15 M (nautical miles). The captain makes a course correction, and, after 4 M, the ship is back on its original course. If the computer had not malfunctioned, the ship would have travelled a shorter distance. What is the difference between the actual distance that the ship travelled and the distance that it would have travelled if it had not gone off course?
CHAPTER 5 TEST

1. During Fitness Awareness Week, data on the number of sit-ups completed by the participants was collected.
   a) Determine the mean and standard deviation for the set of data.
   b) Is the data normally distributed? Explain.
   c) Do you think that a set of data for a different group of participants would be similar? Explain.

<table>
<thead>
<tr>
<th>Number of Sit-ups</th>
<th>Number of Participants</th>
</tr>
</thead>
<tbody>
<tr>
<td>36–40</td>
<td>1</td>
</tr>
<tr>
<td>41–45</td>
<td>5</td>
</tr>
<tr>
<td>46–50</td>
<td>9</td>
</tr>
<tr>
<td>51–55</td>
<td>15</td>
</tr>
<tr>
<td>56–60</td>
<td>23</td>
</tr>
<tr>
<td>61–65</td>
<td>26</td>
</tr>
<tr>
<td>66–70</td>
<td>22</td>
</tr>
<tr>
<td>71–75</td>
<td>14</td>
</tr>
<tr>
<td>76–80</td>
<td>8</td>
</tr>
<tr>
<td>81–85</td>
<td>4</td>
</tr>
<tr>
<td>86–90</td>
<td>2</td>
</tr>
</tbody>
</table>

2. The flight between Vancouver and Winnipeg has a mean time of 156 min, with a standard deviation of 3.5 min. Assuming that the flight times for this trip are normally distributed, determine approximately what percent of the time you could expect the flight time to be
   a) less than 156 min
   b) between 149 min and 156 min
   c) between 152.5 min and 163 min
   d) over 163 min

3. Treena and Maggie both wrote a provincial exam in mathematics. Treena wrote in January, and Maggie wrote in June. Their results are given below.

<table>
<thead>
<tr>
<th>Name</th>
<th>Mark (x)</th>
<th>Provincial Mean (μ)</th>
<th>Provincial Standard Deviation (σ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treena</td>
<td>84%</td>
<td>71%</td>
<td>5.3%</td>
</tr>
<tr>
<td>Maggie</td>
<td>82%</td>
<td>66%</td>
<td>6.2%</td>
</tr>
</tbody>
</table>

   a) Determine which girl’s result is better.
   b) If the results of each exam are normally distributed, what percent of people who wrote the exam in January scored better than Treena?
4. A study of 500 Calgarian taxpayers revealed that 24.1% of these taxpayers make charitable contributions. The study was considered accurate plus or minus 5%, 9 times out of 10. In a particular year, there were 827,120 taxpayers in Calgary.
   a) Determine the projected range of the number of Calgary taxpayers who would make a charitable donation that year.
   b) If the study were conducted using a sample of 1000 Calgarians using the same confidence level, how would the projected range of Calgarians who will make charitable donations be affected? Explain.

5. An Ipsos-Reid study determined that only 28% of online youth consider themselves to be very skilled or expert at using the Internet. According to the study, 24% admit to not being skilled at using the Internet, with the remaining youth identifying themselves as fairly skilled. More than 1200 online interviews, of Canadian youths aged 12 to 17, were completed. Results are considered accurate to within ±2.8%, 19 times out of 20.

   a) Determine the confidence level and confidence interval for the study.
   b) Suppose that there are 68,520 youth, between 12 and 17 years old, in a particular city. State the range of these youth who consider themselves expert at using the Internet.
CHAPTER 6 TEST

1. Tran has made a plan to help him study for his math exam.
   
   - He has broken the material that he has to study into two parts: part A and part B and has, at most, 2 h every evening over the next week to study.
   - He wants to spend at least twice as much time studying for part A as for part B.

   Show graphically all the possible combinations of time that Tran can study for each part. Choose three possible combinations. Explain your choices.

2. These statements describe a solution region for a system of linear inequalities:
   
   - The intersections of its three boundaries are at (0, 4), (6, 1), and (0, −2).
   - The boundary that is farthest left is dashed and vertical, and it intercepts the x-axis at (0, 0).
   - The boundary at the top is solid, with a y-intercept of 4 and a slope of \(-\frac{1}{2}\).
   - The boundary at the bottom is solid, with a slope of \(\frac{1}{2}\) and an x-intercept of 4.
   - The solution region is found in two quadrants.

   Use these statements to identify the system of linear inequalities, and then graph the system.

3. The model below could be used to solve an optimization problem. What points in the feasible region result in the minimum and maximum values of the objective function? Explain how you know.

   **Objective function:**
   \[J = d - f\]

   **Restrictions:** \(d \in I, f \in I\)

   **Constraints:**
   
   \[f \geq 0\]
   \[d + f \leq 8\]
   \[2d - f \geq -5\]
4. Lyle is replacing light bulbs in his apartment.
   • He is using energy-saving bulbs and regular bulbs, and wants to use 21 or fewer bulbs altogether.
   • He wants no more than 18 energy-saving bulbs and at least 2 regular bulbs.
   • The energy-saving bulbs cost $5.95 each, and the regular bulbs cost $7.85 each.

What is the most Lyle can spend on replacement bulbs? How many of each will be used?
CHAPTER 7 TEST

1. For each quadratic, state the direction of opening and the y-intercept.
   a) \( y = 3x^2 + 2x - 5 \)   b) \( y = x^2 - 7x \)   c) \( y = -x^2 + 12 \)

2. A fire hose sprays water in an arch that can be defined by the function
   \[ h(x) = -0.25x^2 + 5x \]
   where \( x \) is the horizontal distance from the hose and \( h(x) \) is the height of the water.
   What is the maximum height of the water, and how far does the water reach?

3. Determine the function that defines the parabola shown.

![Parabola Graph]

4. Solve by graphing.
   a) \( 2x^2 - 7x - 72 = 0 \)   c) \( 2x^3 + 4x - 3 = 5 - 2x - 0.5x^3 \)
   b) \( -0.5x^2 + 4x + 7 = 13 \)   d) \( z(x + 3) - 4 = 2z(4 - z) \)

5. Solve by factoring. Verify each solution.
   a) \( 4r^2 + 36r + 81 = 0 \)   c) \( 12x^2 + 11x - 15 = 0 \)
   b) \( n^2 - 5n - 84 = 0 \)   d) \( 5y^2 + 20y = 2y - 3y^2 - 7 \)

6. Partially factor the quadratic function \( y = 3x^2 - 12x + 8 \) and sketch its graph.

7. A parabola has vertex \((3.6, 9.8)\) and passes through the point \((5.6, 25.4)\).
   Write an equation for the parabola.

8. The roots of a quadratic equation are \( \frac{2}{5} \) and \(-3\).
   a) Determine the quadratic equation in factored form.
   b) Multiply the factors to express the equation in standard form.

9. Solve using the quadratic formula.
   a) \( b^2 - 11b + 24 = 0 \)   c) \( 14z^2 + 21 = 5z^2 + 5 - 24z \)
   b) \( 3p^2 + 5p - 1 = 0 \)   d) \( 5x^2 + 3x + 4 = 0 \)
10. A suspension bridge has a middle span of 356 m. Two large parabolic cables are suspended from a height of 40 m at either end of the bridge, and are 4 m above the roadway in the centre of the bridge. Determine a quadratic function that models one of these cables.

11. A concert promoter’s profit $P(s)$, in dollars, can be modelled by the function

$$P(s) = -8s^2 + 950s - 2250$$

where $s$ is the price of a ticket, in dollars.

a) If the promoter wants to earn a profit of $20\,000$, what should the ticket price be?

b) Is it possible for the promoter to earn a profit of $30\,000$? Explain.
CHAPTER 8 TEST

1. For each of the following, which is the greater unit rate?
   a) $3.25 for 526 mL or $8.45 for 1.2 L
   b) 45 km/h or 30 mph

2. a) Nuri is researching the fuel efficiency of pickup trucks. He found a website that advertised a hybrid pickup truck with a fuel efficiency of 22 mpg for highway driving. He found another website that advertised a pickup truck with a fuel efficiency of 10 km/L for highway driving. Which truck has a better fuel efficiency? Explain how you know.
   b) Describe a different strategy you could have used to determine the better fuel efficiency in part a).

3. a) Draw a graph to represent hiking 500 m from a campground in 10 min, stopping for 5 min, hiking another 400 m away from the campground in 8 min, and then hiking back to the campground along the same trail in 15 min. Where appropriate, assume you are travelling at a constant rate on each part of the trip.
   b) Explain how each slope on your graph represents a rate.

4. Adrienne read that she can burn 236 Cal by bicycling at 10 mph for 1 h or 502 Cal by bicycling at 14 mph for 1 h. How long would she need to bicycle at 10 mph to burn as many Calories as she would by bicycling at 14 mph for 1 h?

5. The Saamis Tipi was built for the 1988 Olympics in Calgary and then moved to Medicine Hat. Its height is 215.0 ft. The diameter at its base is 160.0 ft. A manufacturer wants to make a replica model for a display that is 1.5 ft high.
   a) What scale factor, to the nearest thousandth, would the manufacturer need to use for the model?
   b) What would be the diameter at the base, to the nearest tenth of a foot?

6. A company ships its sports equipment in similar boxes that are two different sizes. The red box is a right rectangular prism with the dimensions 60 cm by 30 cm by 21 cm. The blue box has dimensions that are related by a scale factor of $\frac{2}{3}$.
   a) What is the volume of the red box?
   b) Determine the volume of the blue box, without using the formula for volume.
   c) What ratio compares the surface area of the blue box to the surface area of the red box?
7. The floor plan of the MacBride Museum of Yukon History in Whitehorse is shown below. The distance across the front is 22 m.
   a) Estimate the area of the main building. Explain your strategy.
   b) Estimate the dimensions of the coach house.
   c) Use a scale ratio of 1:90 and your estimated dimensions to draw a floor plan of the coach house. Label the sides with the dimensions.

8. The diameter of Venus is about 12 100 km. The diameter of Mercury is about 4879 km.
   a) What scale factor, to the nearest hundredth, relates the radius of Mercury to the radius of Venus?
   b) What scale factor, to the nearest hundredth, relates the radius of Venus to the radius of Mercury?
   c) Suppose that you knew the surface area of Mercury. What ratio, to the nearest hundredth, could you use to determine the surface area of Venus?
   d) Suppose that you knew the volume of Venus. What ratio, to the nearest hundredth, could you use to determine the volume of Mercury?