Grade 11 Applied Math
Unit 4
Quadratic Functions

Lesson 1: Characteristics of Quadratic Functions
Lesson 2: Applications of Quadratic Functions
Lesson 3: Systems of Equations

Winnipeg Adult Education Centre
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Quadratic Functions Outcomes Overview

These are the outcomes we will be learning in this unit. **Check off** each box once you feel confident with each outcome:

- ☐ I can describe the characteristics of quadratic functions, including end behaviour and y intercept and x intercept(s), domain and range, maximum value, minimum value.
- ☐ I can sketch the graph of a quadratic functions, including the y intercept, x intercepts(s) and one other point or the vertex.
- ☐ I can determine the value of the leading co-efficient and the direction (positive or negative) the function.
- ☐ Given an x value of quadratic function, I can find the y value.
- ☐ I can determine the x intercept(s) and the y intercept of a quadratic function.
- ☐ I can draw a clearly labelled graph of a quadratic function including all intercepts.
- ☐ I can draw a clearly labelled graph for a contextual problem involving a quadratic function.
- ☐ I can solve a contextual problem given the graph of a quadratic function.
- ☐ I can solve a contextual problem given the equation of a quadratic function.
- ☐ I can solve a contextual problem given a data set or table of values for a quadratic function.
- ☐ I can solve a contextual problem given a written/verbal description of a quadratic function.
- ☐ I can solve a system of linear and/or quadratic equations.
- ☐ I can solve a contextual problem modeled by system of equations.

Lesson 1: Characteristics of Quadratic Functions

GOAL:
To describe the characteristics of a quadratic function including:
- Shape
- Vertex
- Leading coefficient
- End Behaviour
- y-intercept
- x-intercept(s)
- Range



The shape formed by water coming out of a water fountain is called a **parabola**. The equations associated with this type of shape are called **quadratic functions**. Quadratic functions have a distinct shape: it's often described as a cup shape or an inverted cup shape.

Example 1: Using the Graphing Calculator to Analyze Functions

Use your graphing calculator to determine which of the following are quadratic functions.

a) $y = .02x^2 - 3$

b) $y = 2x^3 + 1$

c) $y = -(x+3)(x-1)$

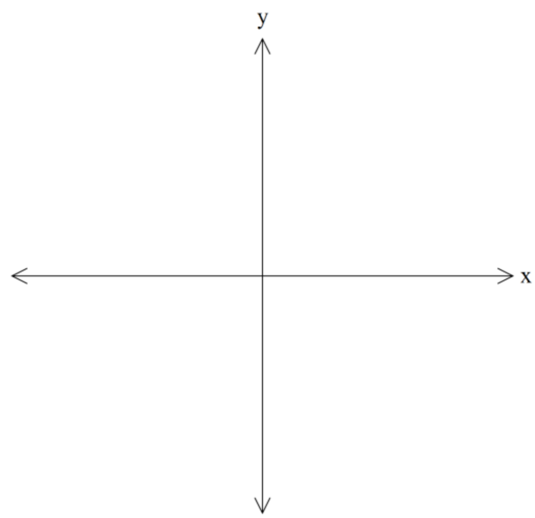
d) $y = 2x - 1$

e) $y = -2x(x^2 + 1)$

f) $y = x(x-5)$

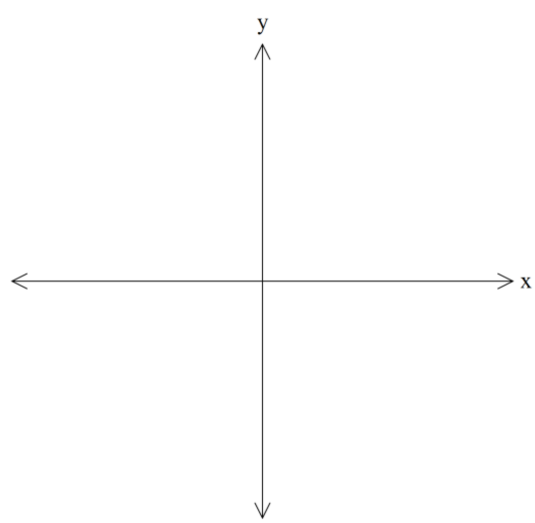
Example 2: Sketching Quadratic Functions using Intercepts

Sketch the graph of $y = x^2 + 2.25x - 7.88$. Label the y intercept and the x intercepts.



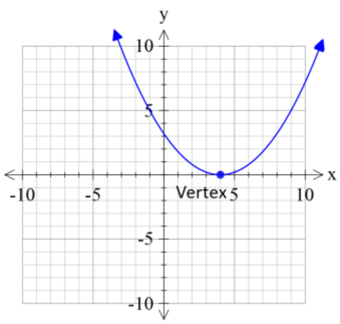
Example 3

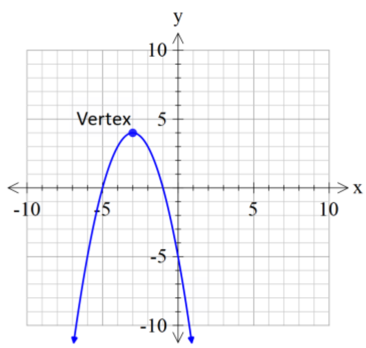
Sketch the graph of $y = -2.65x^2 + 3.9x + 12$. Label the y intercept and the x intercepts.

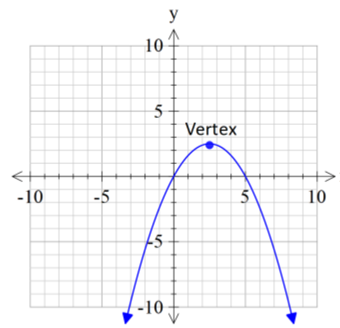


Characteristics of Quadratic Functions

The ***vertex*** of a parabola is the point where the direction of the graph turns (sometimes called the turning point). If you draw a vertical line through the vertex you will see that the parabola is perfectly symmetrical.



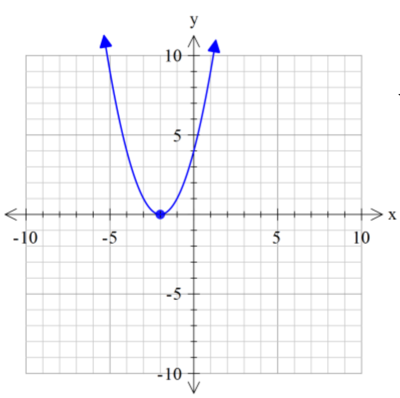




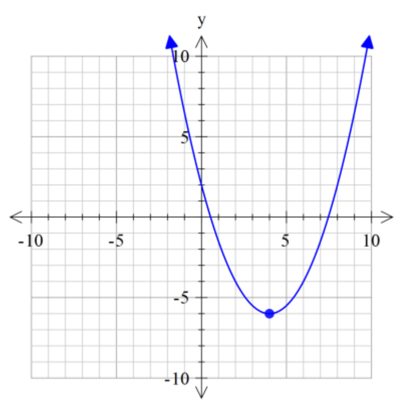
The vertex is an important point of the graph because it is where the **maximum** or **minimum** value occurs. When graphing or analyzing quadratic functions, we should always examine the vertex.

If we know the vertex and the direction of a parabola, then we know a lot about the location of the function.
We can see that the domain of a quadratic function is limitless, like the linear function. But the range of a quadratic function is limited by the location of the vertex and the direction of the opening.

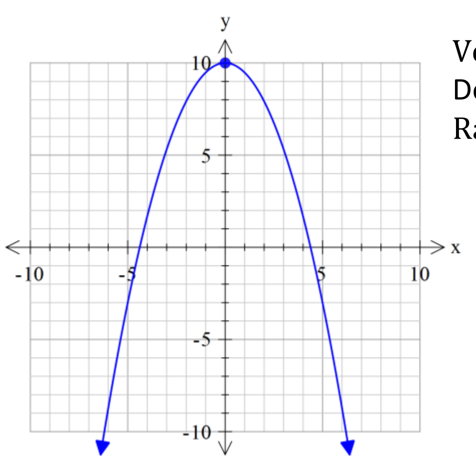
For each of the following graphs, the vertex and range are shown.



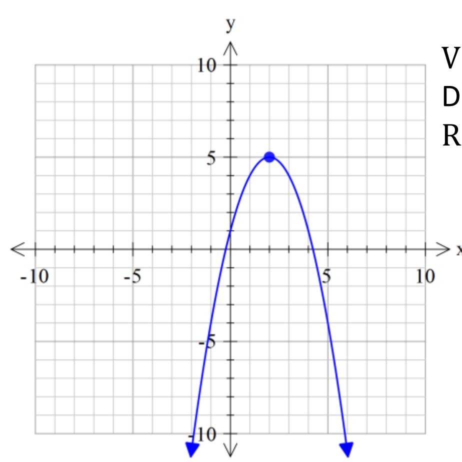
Vertex: $(-2, 0)$
Domain: $x \in R$
Range: $[0, \infty)$



Vertex: $(4, -6)$
Domain: $x \in R$
Range: $[-6, \infty)$

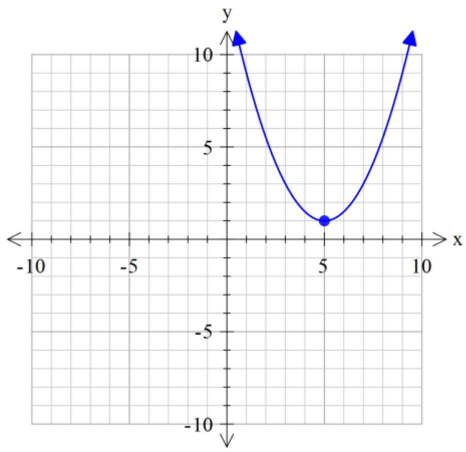


Vertex: $(0, 10)$
Domain: $x \in R$
Range: $(-\infty, 10]$

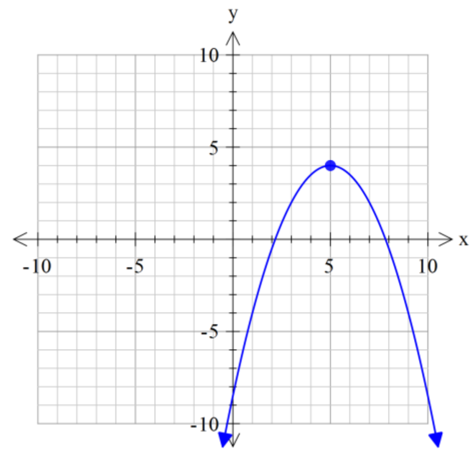


Vertex: $(2, 5)$
Domain: $x \in R$
Range: $(-\infty, 5]$

Example 1: Analyzing Quadratic Functions
State the vertex and the range for the following functions.



Vertex:
Domain:
Range:



Vertex:
Domain:
Range:

In addition to stating the vertex and the domain and range of a quadratic function, many other characteristics can be described such as end behaviour and x and y intercepts.

The quadratic function shown below can be described as having the following characteristics:

Shape: Parabola opening upward.

Sign of leading coefficient: Positive

End Behaviour: Q II to Q I.

y-intercept: 0.

x-intercepts: -4 and 0.

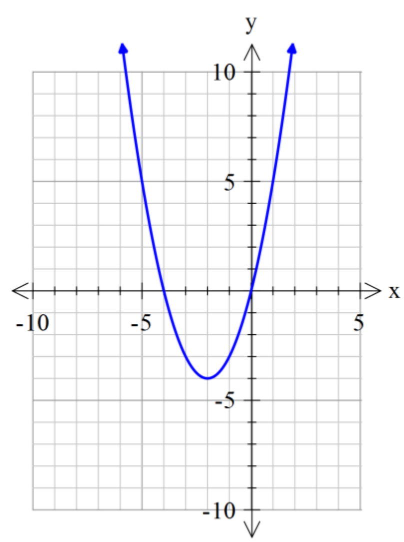
Vertex: $(-2, -4)$

Minimum Value: -4

Maximum Value: None

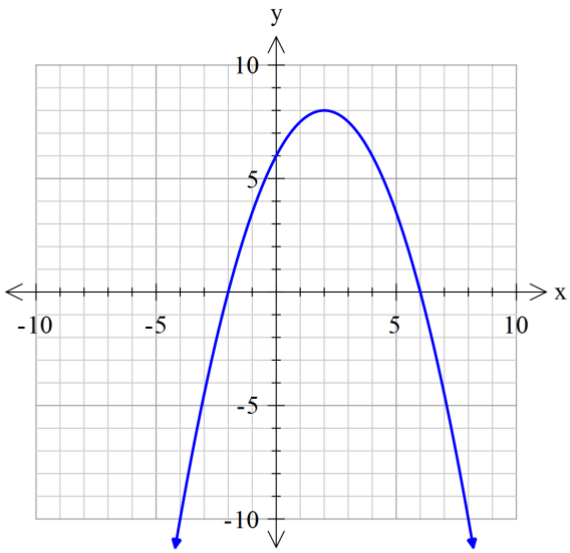
Domain: $x \in R$

Range: $[-4, \infty)$



Using the vocabulary from Example 1, fill in the blanks for the quadratic functions in Examples 2 and 3.

Example 2



Direction of opening ________

Sign of leading co-efficient ________

End behaviour _____________

y-intercept _____________

x-intercept(s) _____________

Vertex _____________

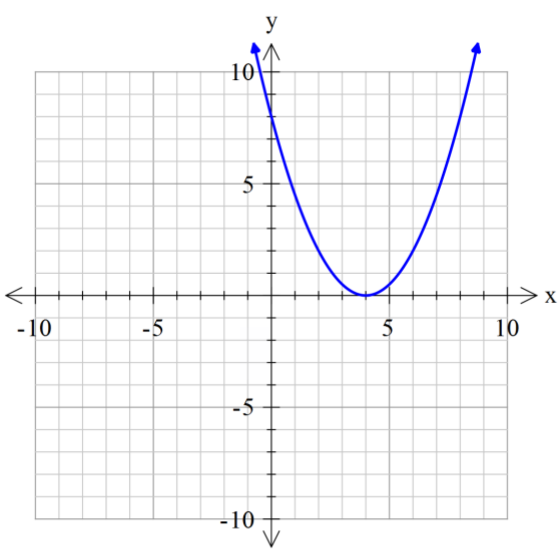
Minimum value _____________

Maximum value _____________

Domain _____________

Range _____________

Example 3



Direction of opening ________

Sign of leading co-efficient ________

End behaviour _____________

y-intercept _____________

x-intercept(s) _____________

Vertex _____________

Minimum value _____________

Maximum value _____________

Domain _____________

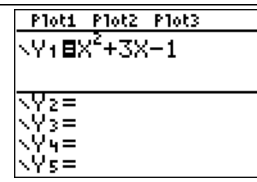
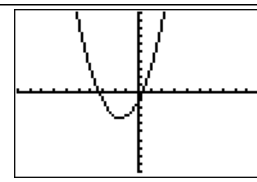
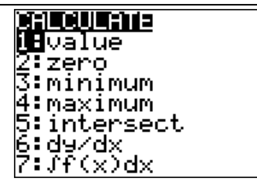
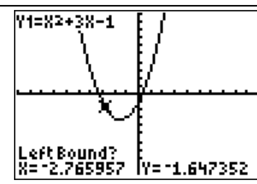
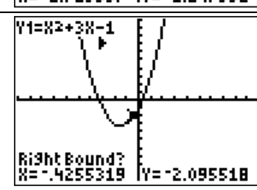
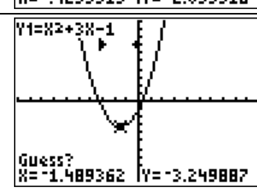
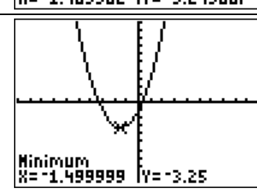
Range _____________

Using the Graphing Calculator to Determine the Vertex

We already know how to graph a function and find the intercepts using the calculator. We also need to know how to determine the vertex. The following example shows how to use the calculator to find the vertex of a quadratic function.

Example 4:

Determine the vertex of $y = x^2 + 3x - 1$

Instructions	**What you should see**
Enter the equation and press GRAPH.	
Note whether the parabola opens *up* or *down*. (This one opens *up*.)	
Use 2nd TRACE to access the CALC menu.	
If the parabola opens upward, choose **3: minimum** If the parabola opens downward, choose **4: maximum**	
Notice that your screen asks: Left Bound? A message like this indicates that you should move your cursor to the LEFT side of the vertex of the parabola. Once you have moved the cursor anywhere to the left of the vertex you can press ENTER.	
The next screen asks: Right Bound? Move your cursor to the RIGHT of the vertex and press ENTER.	
The final question asks: Guess? Move your cursor *close* to the vertex and press ENTER.	
Conclusion: The vertex is $(-1.5, -3.25)$. We also say that the *minimum* value of -3.25 occurs when $x = -1.5$.	

Example 5: Using the Graphing Calculator to determine Characteristics of Quadratic Functions

Analyze each quadratic function by providing the requested characteristics.

a) $y = 0.5x^2 - 2x + 7$

Direction of opening: _______

Sign of leading co-efficient: _______

End behaviour: ___________

y-intercept: ___________

x-intercept(s): ___________

Vertex: ___________

Minimum value: ___________

Maximum value: ___________

Domain: ____________

Range: ___________

b) $y = (7 - 2x)(x + 10)$

Direction of opening: _______

Sign of leading co-efficient: _______

End behaviour: ___________

y-intercept: ___________

x-intercept(s): ___________

Vertex: ___________

Minimum value: ___________

Maximum value: ___________

Domain: ____________

Range: ___________

c) $y = -34.2x^2 + 8.25x - 44$

Direction of opening: _______

Sign of leading co-efficient: _______

End behaviour: ___________

y-intercept: ___________

x-intercept(s): ___________

Vertex: ___________

Minimum value: ___________

Maximum value: ___________

Domain: ____________

Range: ___________

Assignment 1

Analyze each quadratic function by sketching (if required) and providing the requested characteristics.

1. Direction of opening: ________

   Sign of leading co-efficient: ________

   End behaviour: ____________

   y-intercept: ____________

   x-intercept(s): ____________

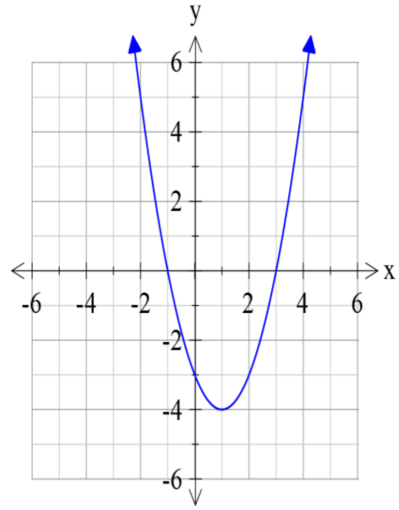
   Vertex: ____________

   Minimum value: ____________

   Maximum value: ____________

   Domain: ______________

   Range: ______________



2. $y = 3x^2 + 10x - 13$

   Direction of opening: ________

   Sign of leading co-efficient: ________

   End behaviour: ____________

   y-intercept: ____________

   x-intercept(s): ____________

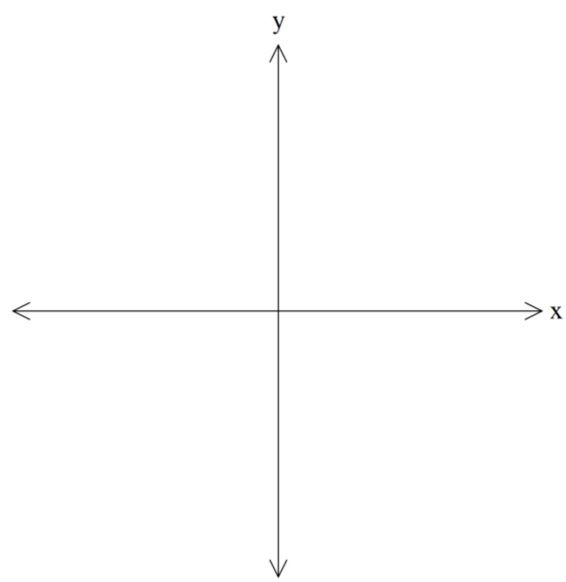
   Vertex: ____________

   Minimum value: ____________

   Maximum value: ____________

   Domain: ______________

   Range: ______________



3. $y = -0.5(x-1)^2 + 7$

   Direction of opening: _______

   Sign of leading co-efficient: _______

   End behaviour: ___________

   y-intercept: ___________

   x-intercept(s): ___________

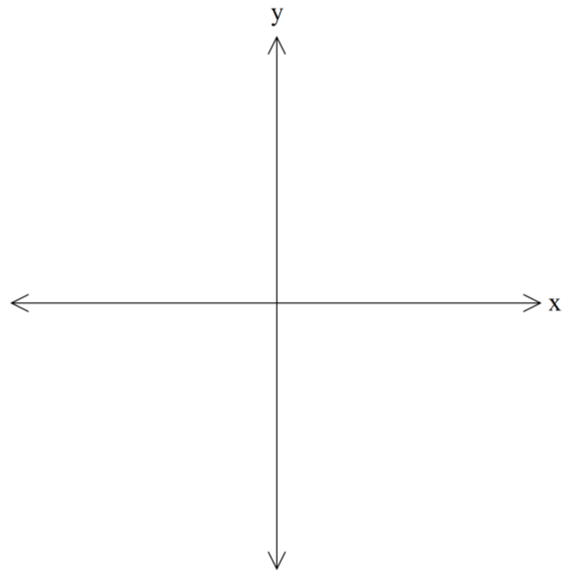
   Vertex: ___________

   Minimum value: ___________

   Maximum value: ___________

   Domain: _____________

   Range: _____________



4. $y = -2x^2 + 6x + 17$

   Direction of opening: _______

   Sign of leading co-efficient: _______

   End behaviour: ___________

   y-intercept: ___________

   x-intercept(s): ___________

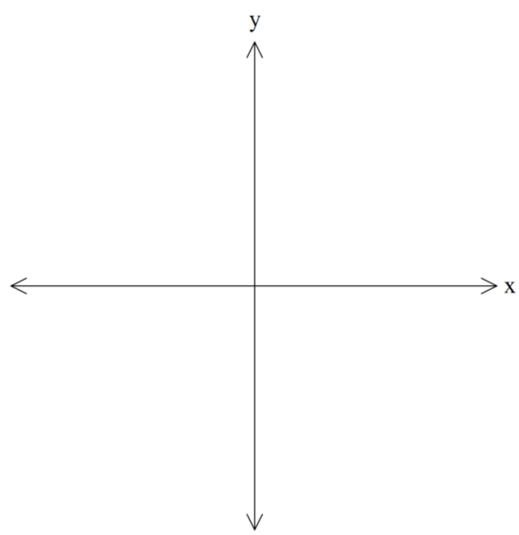
   Vertex: ___________

   Minimum value: ___________

   Maximum value: ___________

   Domain: _____________

   Range: _____________



5. $y = -12x^2 + 192x - 543$

Direction of opening: _______

Sign of leading co-efficient: _______

End behaviour: ___________

y-intercept: ___________

x-intercept(s): ___________

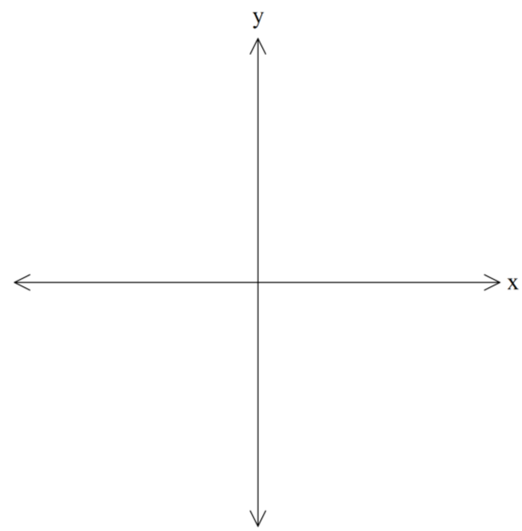
Vertex: ___________

Minimum value: ___________

Maximum value: ___________

Domain: _____________

Range: _____________



6. $y + 3 = 5x + x^2$

Direction of opening: _______

Sign of leading co-efficient: _______

End behaviour: ___________

y-intercept: ___________

x-intercept(s): ___________

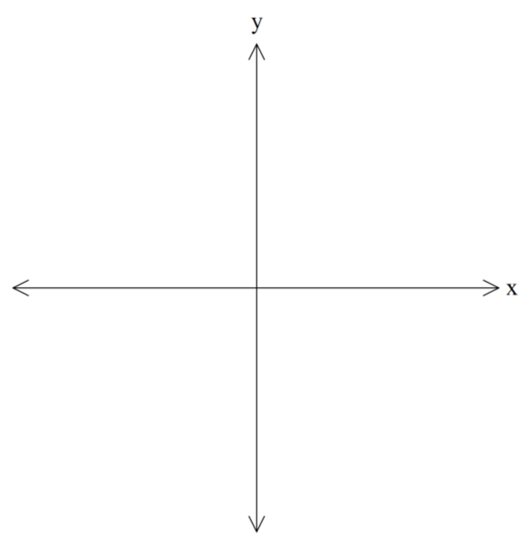
Vertex: ___________

Minimum value: ___________

Maximum value: ___________

Domain: _____________

Range: _____________



7. $5x^2 - 32x + 51 = y$

Direction of opening: ________

Sign of leading co-efficient: ________

End behaviour: ____________

y-intercept: ____________

x-intercept(s): ____________

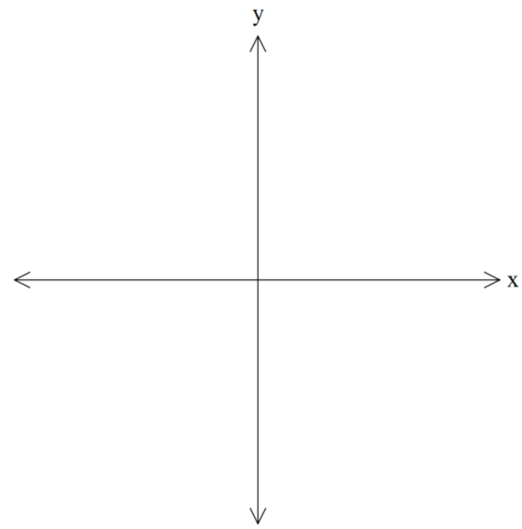
Vertex: ____________

Minimum value: ____________

Maximum value: ____________

Domain: ______________

Range: ______________



8. $x^2 + 17x - 23 = y$

Direction of opening: ________

Sign of leading co-efficient: ________

End behaviour: ____________

y-intercept: ____________

x-intercept(s): ____________

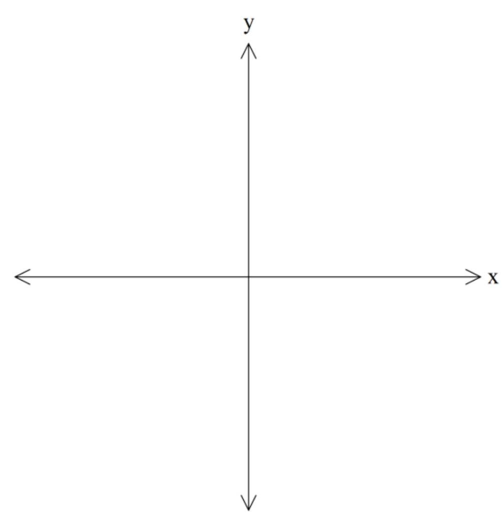
Vertex: ____________

Minimum value: ____________

Maximum value: ____________

Domain: ______________

Range: ______________



Lesson 2: Applications of Quadratic Functions

Example 1: Quadratic Application with Graph Given

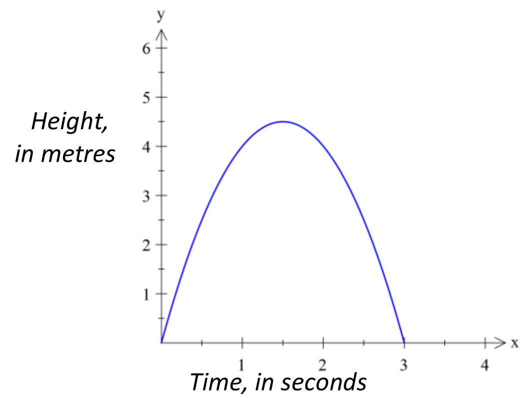
A ball is kicked into the air. The graph below shows the height of the ball versus time.

a) How long (how many seconds) is the ball in the air?

b) What is the maximum height attained?

c) How long does it take the ball to reach its maximum height?

d) What is the minimum height of the ball?



Example 2: Quadratic Application with Equation Given

You are playing golf on the Moon. You hit a ball with your golf club. The height of the ball, H, in metres, with respect to time, t, in seconds, can be modelled by the equation

$$H = -0.8t^2 + 40t$$

a) Sketch the graph. Show the vertex and intercepts.

b) What is the maximum height of the ball?

c) How long (for how many seconds) was the ball in the air?

Example 3: Quadratic Application with Equation Given

A water arch at a splash pad is defined by the following function:

$$V = -0.15x^2 + 3x$$

Where x represents the horizontal distance, in feet, from the opening in the ground, and V represents the vertical distance, in feet.

a) What is the maximum height of the arch of the water?

b) How far from the opening in the ground can the water reach?

Example 4: Quadratic Application with Table of Values Given

a) An archway is parabolic in shape. Some measurements taken from the archway are shown in the table below. Determine the quadratic regression equation.

Horizontal Distance (ft)	0	2	4	6
Height (ft)	0	8.59	9.38	2.34

b) What is the maximum height of the archway?

c) What is the minimum height?

d) How wide is the archway at ground level?

Example 5: Quadratic Application with Table of Values

A ball was thrown into the air and the path generated the following data:

Time (sec)	0	0.25	0.5	0.75	1	1.25	1.5	1.75
Height (m)	1	9	11	13	12	11	7	0.5

a) Determine the quadratic regression equation that best matches this data.

b) What is the maximum height the ball reaches, and when does it reach this maximum height?

c) How high will the ball be after 0.6 sec?

d) How long does it take until the ball hits the ground at the end of the throw?

e) How long does it take before the ball reaches 5 m in height for the first time?

Example 6: Quadratic Application with Verbal Description Given

A company that sells jeans finds that when the jeans are priced at $80 per pair, they can sell 500 pairs. It is estimated that for each $2.00 decrease in price, the company can sell 50 more pairs of jeans.

a) Complete the following table of values.

b) Determine the quadratic regression equation that models the revenue as a function of the price.

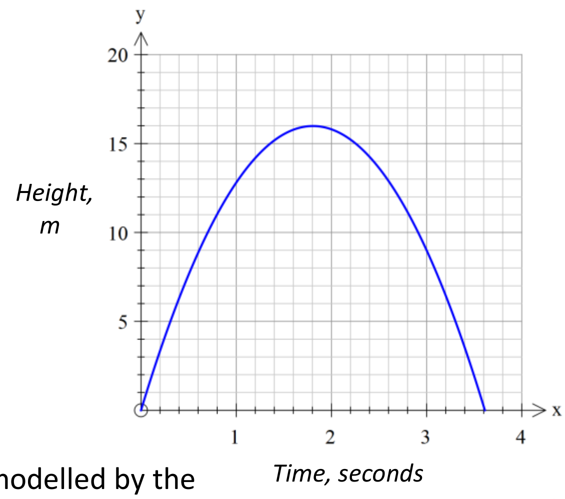
c) Find the price of jeans that will generate the maximum revenue.

d) Determine the maximum revenue.

Number of Pairs Sold	Price	Revenue $
500	80	40 000

Assignment 2

1. The graph at the right show the height of a football versus time.
   a) What was the maximum height attained by the football?
   b) How many seconds did it take for the football to reach its maximum height?
   c) For how long was the football in the air?



2. A grasshopper takes a jump in a motion that can be modelled by the equation $y = -49x^2 + 160x$ where y is the height, in centimetres, and x is the time, in seconds.
   a) What does the y-intercept represent?
   b) What is the grasshopper's maximum height?
   c) How many seconds does it take for the grasshopper to land on the ground?
   d) What is the height of the grasshopper 1 second after it jumps?

3. A gymnast is jumping on a trampoline. His height, y, in metres, above the floor on each jump is roughly approximated by the function $\mathrm{y} = -4.9\mathrm{x}^2 + 5.3\mathrm{x} + 1.2$ where x represents the time, in seconds, since he left the trampoline. Determine his maximum height on each jump.

4. Olympic shot-put competitors throw a heavy metal ball called a shot. The data for one such throw is measured and the results are as follows:

Time (sec)	0	0.1	0.2	0.3	0.4	0.5	0.6
Height (m)	1.4	2	2.2	2.3	2.2	1.5	0.3

   a) Find the quadratic regression equation for this data.
   b) What is the maximum height the shot reaches?
   c) When does it reach this maximum height?
   d) When does the shot hit the ground at the end of its flight?

5. One of the water fountains at Portage Place mall shoots a plume of water into the air at regular intervals. The following data was recorded:

Time (sec)	0	0.5	1	1.5	2	2.5	3
Height (m)	–1	4	9	11	11	8	–1

a) Find the quadratic regression equation for this data.
b) What is the maximum height reached by the water?
c) How long does it take until it reaches this maximum height?
d) How much time does it take until you can actually *see* the water for the first time?
e) For how many seconds is the water visible to bystanders?
f) When does the water reach a height of 10 m for the first time?

6. The manager of a bike store is setting the price for a new model. Based on past sales, he predicts that he can sell 280 bikes if the price is $360. For every $10 increase in price, he expects to sell five fewer bikes.

a) Complete the following table of values.

Price $	Number of Bikes Sold	Revenue $
360	280	100 800

b) Determine the quadratic regression equation that models the revenue as a function of the price.
c) What price will yield the maximum revenue?
d) What is the maximum amount of revenue?

Lesson 3: Solving Systems of Equations

GOAL:
Model and solve problems that involve systems of equations.

A system of equations is when we have a set of two or more equations of functions. A system of equations can consist of different types of functions. For example, a system can consist of linear equations, quadratic equations, or a mixture of both. We are usually interested in where the equations of a system intersect each other. If they intersect, they have some points in common. Therefore, to solve a system of equations, we find the intersection points.

Example 1

Solve the system given by the following equations:

$$y = -2(x-3)^2 + 6 \quad \text{and} \quad y = -4x + 15$$

Example 2

Find the possible meeting points if one boat is travelling according to the equation $y = 6x + 4$ and a second boat is travelling according to the equation $y = 3x^2 + 2x - 5$.

Example 3

Determine the potential crash points for two aircraft following the following trajectories.

Aircraft 1: $y = 3x^2 + 4$ Aircraft 2: $y = (x-5)(1-x)$

Assignment 3

1. Solve the following system of equations:

$$y = 0.5x^2$$
$$y = -\frac{3}{2}x + \frac{7}{2}$$

   Sketch the graph of the system and state the solution(s).

2. Find the point(s) of intersection for

$$y = x^2 - 2$$
$$y = -2x^2 + 6x + 5.$$

   Draw the graph of your solution.

3. A baseball is thrown. The height of the baseball is given by the equation

$$y = -5(t-2)^2 + 35$$

   where y is the height in meters and t is the time in seconds. A paintball is shot at the baseball following a trajectory of $y = \frac{1}{3}(25x + 15)$. Will the paintball hit the baseball? At what time?

4. A pelican flying in the air over water drops a crab from a height of 30 feet. The height of the crab over the water as it falls can be represented by the function $h = -16t^2 + 30$, where h is the height of the crab in feet, and t is time, in seconds. To catch the crab as it falls, a seagull flies along a path represented by the function $y = -8t + 15$. Can the seagull catch the crab before the crab hits the water? Justify your answer.

5. Suppose the annual demand, ***q*** million cars, for a front-wheel drive economy car is related to its production price, ***p*** thousand dollars, by the equation: $p = 9 - 2q^2$.
   At a production price of ***p*** thousand dollars, the manufacturers are willing to build ***q*** million cars a year according to the equation: $q^2 + 5q + 1 = p$.
   a) The intersection of the two equations gives the "equilibrium point". Find the equilibrium point for these equations.
   b) Determine the equilibrium production price of the car.

Answer Key

Assignment 1

1. Direction of opening up
   Sign of leading co-efficient positive
   End behaviour Q ll to Q l
   y intercept -3
   x intercept(s) $-1 \text{ and } 3$
   Vertex $(1, -4)$
   Minimum value -4
   Maximum value no maximum
   Domain $(-\infty, \infty)$
   Range $[-4, \infty)$

2. Direction of opening up
   Sign of leading co-efficient positive
   End behaviour Q ll to Q l
   y intercept -13
   x intercept(s) $-4.33 \text{ and } 1$
   Vertex $(-1.67, -21.33)$
   Minimum value -21.33
   Maximum value no maximum
   Domain $(-\infty, \infty)$
   Range $[-21.33, \infty)$

3. Direction of opening down
   Sign of leading co-efficient negative
   End behaviour Q lll to Q lV
   y intercept 6.5
   x intercept(s) $-2.74 \text{ and } 4.74$
   Vertex (1, 7)
   Minimum value no minimum
   Maximum value 7
   Domain $(-\infty, \infty)$
   Range $(-\infty, 7]$

4. Direction of opening — down
   Sign of leading co-efficient — negative
   End behaviour — Q lll to Q lV
   y intercept — 17
   x intercept(s) — -1.78 *and* 4.78
   Vertex — (1.5, 21.5)
   Minimum value — *no* minimum
   Maximum value — 21.50
   Domain — $(-\infty, \infty)$
   Range — $(-\infty, 21.50\,]$

5. Direction of opening — down
   Sign of leading co-efficient — negative
   End behaviour — Q lll to Q lV
   y intercept — -543.00
   x intercept(s) — 3.67 *and* 12.33
   Vertex — (8, 225)
   Minimum value — *no* minimum
   Maximum value — 225
   Domain — $(-\infty, \infty)$
   Range — $(-\infty, 225\,]$

6. Direction of opening — up
   Sign of leading co-efficient — positive
   End behaviour — Q ll to Q l
   y intercept — -3
   x intercept(s) — -5.54 *and* 0.54
   Vertex — $(-2.5, -9.25)$
   Minimum value — -9.25
   Maximum value — no maximum
   Domain — $(-\infty, \infty)$
   Range — $[-9.25, \infty\,)$

7. Direction of opening up
   Sign of leading co-efficient positive
   End behaviour Q II to Q I
   y intercept 51
   x intercept(s) $3\ and\ 3.4$
   Vertex $(3.2, -0.2)$
   Minimum value -0.2
   Maximum value no maximum
   Domain $(-\infty, \infty)$
   Range $[-0.2, \infty)$

8. Direction of opening up
   Sign of leading co-efficient positive
   End behaviour Q II to Q I
   y intercept -23
   x intercept(s) -18.26 *and* 1.26
   Vertex $(-8.5, -95.25)$
   Minimum value -95.25
   Maximum value no maximum
   Domain $(-\infty, \infty)$
   Range $[-95.25, \infty)$

Assignment 2

1. a) 16 m b) 1.8 seconds c) 3.6 seconds
2. a) the initial height of the grass hopper b) 130.61 cm c) 3.27 seconds d) 124 cm
3. 2.63 m
4. a) $y = -16.55x^2 + 8.39x + 1.33$ b) 2.40 m c) 0.25 seconds d) 0.63 seconds

5. a) $y = -5.42x^2 + 17.00x - 2.00$ b) 11.31 m c) 1.57 seconds d) 0.12 seconds
   e) 2.89 seconds f) 1.07 seconds

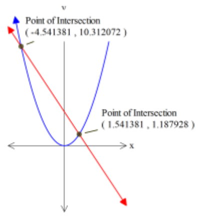
6. a) The missing values are:

370	275	101750
380	270	102600
390	265	103350

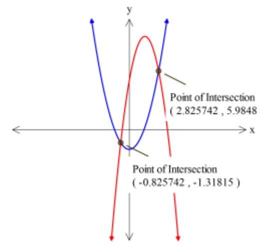
   b) $y = 0.5x^2 + 460x$
   c) $460 d) $105 800

Assignment 3

1. (-4.54, 10.31) and (1.54, 1.19)



2. (-0.83, -1.32) and (2.83, 5.98)



3. Yes, it will hit, at 3 seconds (30 m in the air).
4. Yes it can – it will catch the gull after 1.25 sec (5 feet in the air).
5. a) annual demand of 1 million cars b) the production price is $7000

Quadratic Functions Outcomes Summary

These are the outcomes that have been covered in this unit. **Check off** each box if you are confident that you can demonstrate that skill:

- ☐ I can describe the characteristics of quadratic functions, including end behaviour and y intercept and x intercept(s), domain and range, maximum value, minimum value.
- ☐ I can sketch the graph of a quadratic functions, including the y intercept, x intercepts(s) and one other point or the vertex.
- ☐ I can determine the value of the leading co-efficient and the direction (positive or negative) the function.
- ☐ Given an x value of quadratic function, I can find the y value.
- ☐ I can determine the x intercept(s) and the y intercept of a quadratic function.
- ☐ I can draw a clearly labelled graph of a quadratic function including all intercepts.
- ☐ I can draw a clearly labelled graph for a contextual problem involving a quadratic function.
- ☐ I can solve a contextual problem given the graph of a quadratic function.
- ☐ I can solve a contextual problem given the equation of a quadratic function.
- ☐ I can solve a contextual problem given a data set or table of values for a quadratic function.
- ☐ I can solve a contextual problem given a written/verbal description of a quadratic function.
- ☐ I can solve a system of linear and/or quadratic equations.
- ☐ I can solve a contextual problem modeled by system of equations.

Ongoing Self-Assessment for Mathematics Students

Understanding
How confident are you in your ability to demonstrate understanding of the outcomes of this unit?

My ability to demonstrate understanding is a: ***STRENGTH*** ☐ ***CHALLENGE*** ☐

Attendance
Did you have consistently good attendance during this unit?

My attendance is a: ***STRENGTH*** ☐ ***CHALLENGE*** ☐

Out of Class Practice
Did you feel that when you needed to practice a math skill outside of class, you were able to do so?

My ability to practice outside of class time is a: ***STRENGTH*** ☐ ***CHALLENGE*** ☐

Accessing Help
If you answered ***CHALLENGE*** to any of the questions above, consider the following options for accessing help in order to be more successful in this course:

- Talk to your ***TEACHER***.
- Make time to visit the ***RESOURCE ROOM (ROOM 104).***
- Get help / support / materials from a ***CLASSMATE.***
- Use any resources provided on a ***CLASS BLOG*** (if available).

You have completed a unit in this Math course. Please take some time to reflect on your thoughts regarding your academic strengths and challenges as they relate to the outcomes of this unit. You can also reflect on any previous outcomes of this course.

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