Grade 11 Applied Math
Unit Three
Introduction to Graphing & Linear Functions

Lesson 1: The Cartesian Plane
Lesson 2: Displaying Data and Describing Trends
Lesson 3: Using Technology to Display Data and find a Regression Equation
Lesson 4: Identifying Characteristics of Linear Functions from a Given Equation
Lesson 5: Finding the Intercepts of a Linear Function
Lesson 6: Applications of Linear Functions

Winnipeg Adult Education Centre
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Linear Functions Outcomes Overview

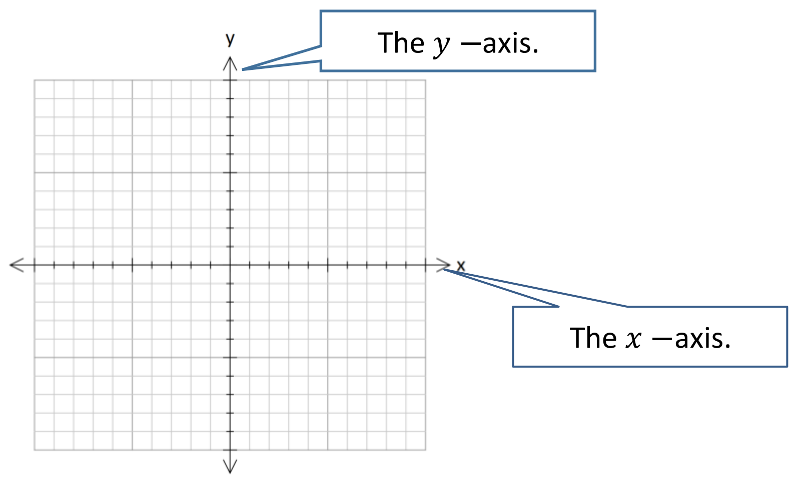
These are the outcomes we will be learning in this unit. **Check off** each box once you feel confident with each outcome:

- ☐ I can describe the characteristics of linear function, including end behaviour and y intercept and x intercept(s), domain and range, maximum value, minimum value.
- ☐ I can sketch the graph of a linear function, including labelling the y intercept, x intercepts(s) and one other point or the vertex.
- ☐ I can determine the value of the leading co-efficient and the direction (positive or negative) the function.
- ☐ Given an x value of a linear function, I can find the y value.
- ☐ I can determine the x intercept(s) and the y intercept of a linear function.
- ☐ I can draw a clearly labelled graph of a linear function including all intercepts.
- ☐ I can draw a clearly labelled graph for a contextual problem involving a linear function.
- ☐ I can solve a contextual problem given the graph of a linear function.
- ☐ I can solve a contextual problem given the equation of a linear function.
- ☐ I can solve a contextual problem given a data set or table of values for a linear function.
- ☐ I can determine whether a function is linear.
- ☐ I can solve a contextual problem given a written/verbal description of a linear function.

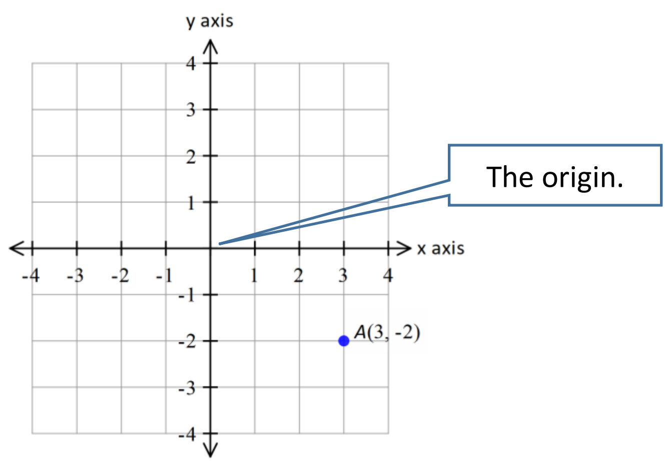
Lesson 1: The Cartesian Plane

Goal: To develop an understanding of the Cartesian Co-ordinate System including x axis, y axis, quadrants, and co-ordinates.

To locate the position of a particular point on a plane, a co-ordinate system is often used. In mathematics we use the Cartesian Plane (sometimes called the Cartesian coordinate system). In this system the horizontal number line is usually the x-axis and the vertical number line is usually called the y-axis.

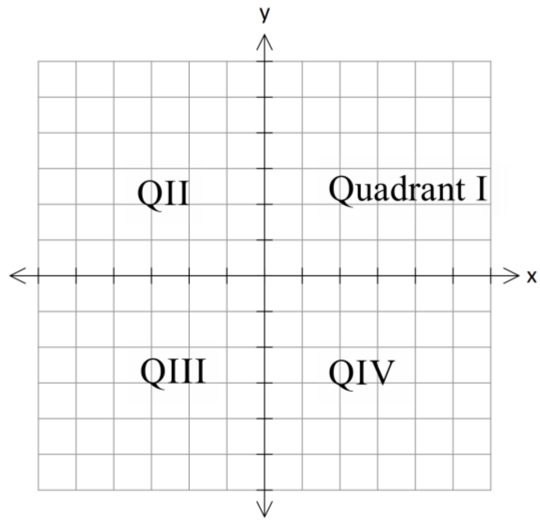


Points are located by giving two co-ordinates that give the value of the x −coordinate and the y −coordinate. In the diagram below, point A has co-ordinates $(3, -2)$.



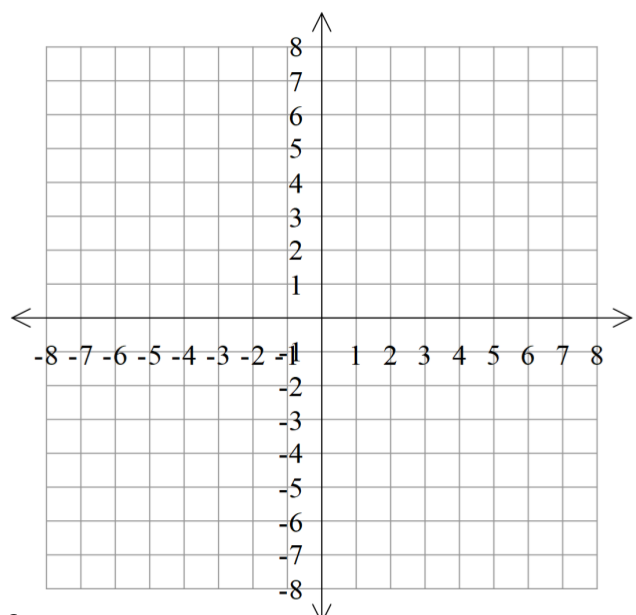
When we write the point, the first coordinate is always the x − coordinate and the second coordinate is always the y − coordinate. This is why it is called an *ordered* pair. The two coordinates are placed in brackets and separated by a comma.
The point where the x axis and y axis meet is called the ***origin.*** The origin represents a position of zero on both axes, and therefore the co-ordinates of the origin are (0, 0).

The Cartesian plane has four sections called quadrants. The quadrants are named using Roman Numerals; Quadrant I is the upper right quadrant and they progress counter clockwise to quadrants II, III and IV. Points may lie on the origin, the x – axis, the y – axis, or in any one of the quadrants.



Example 1

a) Plot the following points on the coordinate system below.
A (2, 5)
B (−3, 1)
C (−3, −5)
D) (7, −2)
E (0, 3)
F (−5, 0)

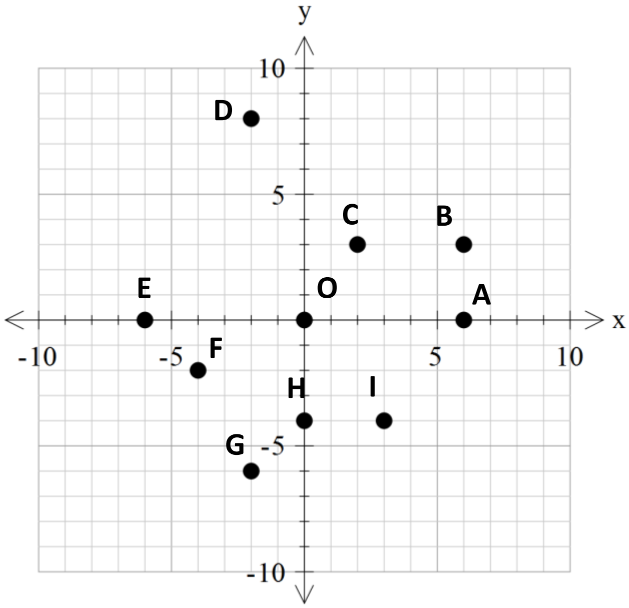


b) Which point above is located in Quadrant IV?

c) Which point is located on the x – axis?

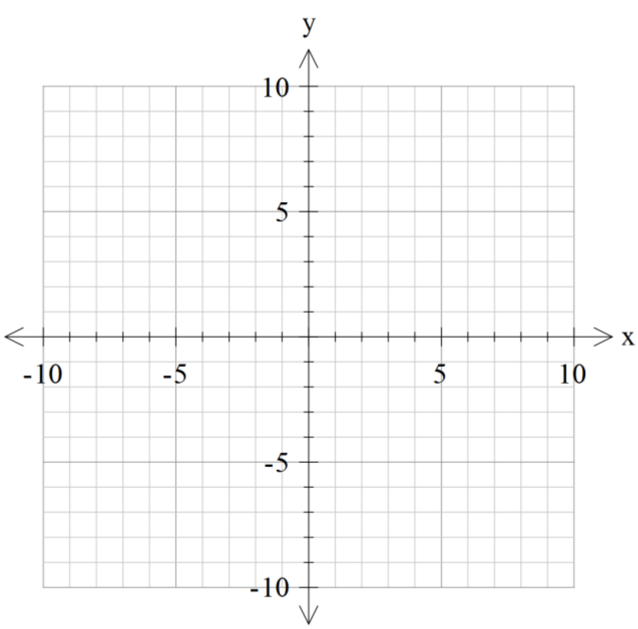
In-Class Activity: Plotting Points

1. Write the coordinates of each point shown on the grid below.



A:
B:
C:
D:
E:
F:
H:
I:
O:

2. Plot the following points on the grid below. Label each point.
   A $(3,5)$ B $(0,4)$ C $(-3,-2)$ D $(-1,6)$ E $(-2,0)$ F $(3,-6)$ G $(5,0)$ H $(0,-3)$



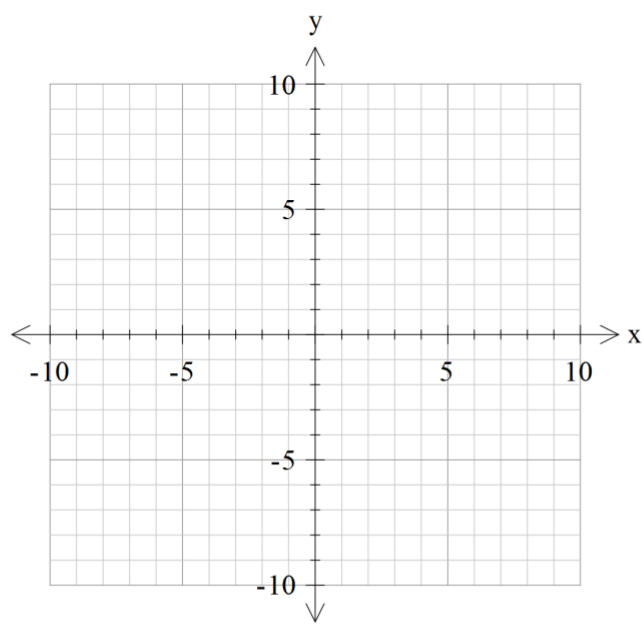
3. In which quadrant, or on which axis, do each of the following points lie?

a) $(3, -5)$ b) $(-6, -2)$ c) $(5, 7)$ d) $(-2, 4)$ e) $(-5, -3)$

f) $(4, 6)$ g) $(-8, 5)$ h) $(0, 4)$ i) $(-5, -2)$ j) $(3, -1)$

4. Plot the following on the grid provided.

x	y
0	9
2	7
-4	5
6	3
8	-1
0	-5
-5	-7
6	0

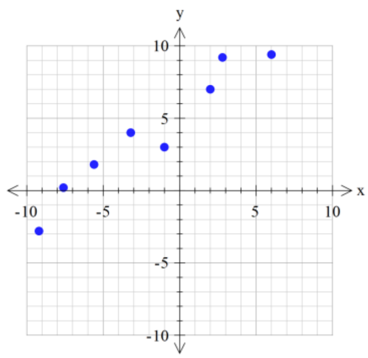


Lesson 2: Displaying Data and Describing Trends

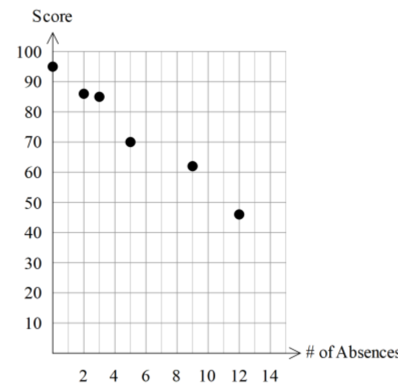
Goals:
- To create and interpret scatterplots that result in a linear pattern.
- To determine and use appropriate scales.

Examining Data From a Scatterplot

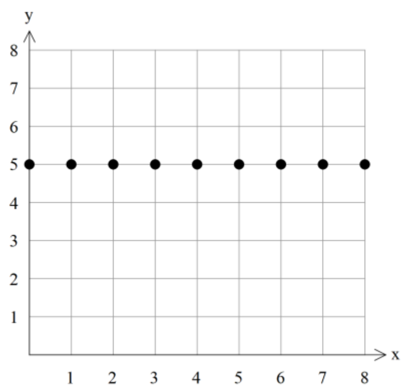
Sometimes the data that is plotted displays certain patterns. These patterns can be described in different ways. In this unit we will focus on linear patterns. The word 'line' is contained in the word 'linear' and we will see that our data in this unit will resemble the shape of a line.



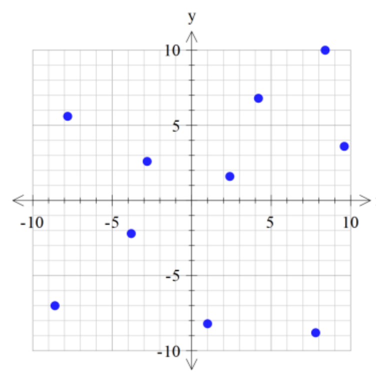
The points are arranged in the shape of a line pointing upwards (when we 'read' the graph from left to right).
We say that this graph shows an upward trend. Generally, as the x value increases, the y value increases. We say that there is a **positive correlation** between x and y.



This graph shows a downward trend. Generally, as the x value increases, the y value decreases. As the number of absences increases, the score decreases. We say that there is a **negative correlation** between the number of absences (the x values) and the score (the y values).



This graph shows that as the x value increases, there is no change in the y value. We call this a **constant** relation.

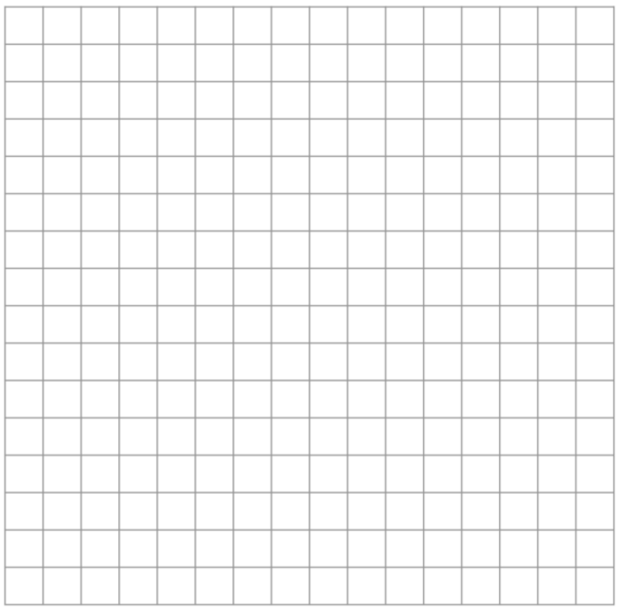


This graph shows a random pattern. There does not appear to be a correlation between the two variables. This say there is **no correlation**.

Graphing Linear Data

Example 1

a) Plot the graph of the data provided in the table. The table provides data regarding the number of absences per student and the student's final mark.
Be sure to include your axes, labels, and a scale on each axis.



# of Absences	Student Mark (%)
8	65
3	90
5	85
4	62
0	87
2	97

Choosing Appropriate Scales

To choose appropriate scales for your axes, consider:

What is the minimum x value?

What is the maximum x value?

What is the scale for your x axis?

What is the minimum y value?

What is the maximum y value?

What is the scale for your y axis?

Notice that in any cases, negative numbers are not needed. In these cases, we just need Quadrant ! to graph the data.

b) Examine the graph of the data that you plotted. Explain what the data is telling us specifically about the correlation between the # of Absences and the Student Mark?

Example 2

a) Plot the graph of the data provided in the table. The table provides data about the speed of a vehicle and its stopping distance after the brakes are applied.
Be sure to include your axes, labels, and a scale on each axis.

Speed (km/h)	Distance before stopping (metres)
50	14
80	36
60	20
110	68
40	9
30	5
150	126

Remember, to choose appropriate scales, consider:

What is the minimum x value?

What is the maximum x value?

What is the scale for your x axis?

What is the minimum y value?

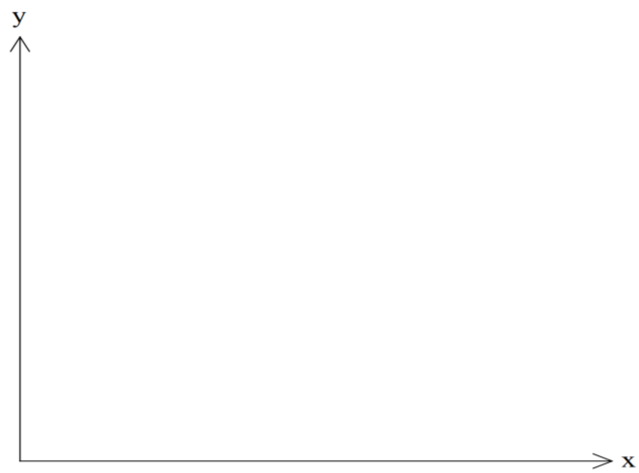
What is the maximum y value?

What is the scale for your y axis?

b) Examine the graph of the data that you plotted. Explain what the data is telling us specifically about the correlation between the speed of a vehicle and its stopping distance?

Example 3

a) This table shows the numbers of kilometers per hour over the speed limit and the amount of fine.

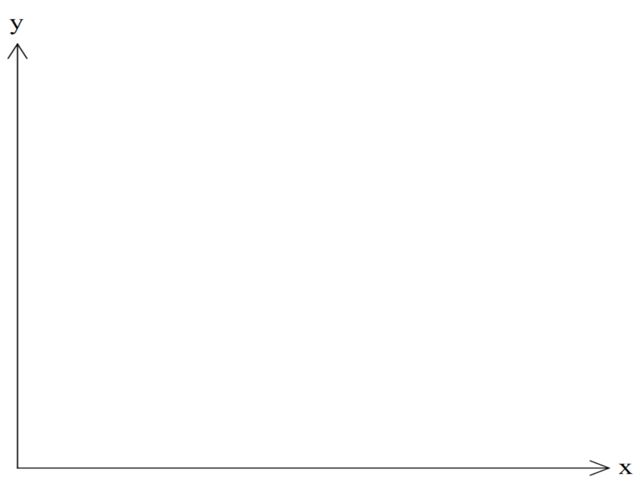


# of Km/hr over the speed limit	Fine ($)
50	190
10	90
25	105
5	85

b) Explain what the data is telling us specifically about the correlation between the number of km/hr over the speed limit and the amount of fine.

Example 4

a) For a chemistry experiment, different volumes of ethanol are poured into a beaker. After each volume is poured into the beaker, the beaker is weighed. Here are the results.



Volume (mL)	Mass of beaker and ethanol (g)
0	90
100	168
50	129
150	207
200	246

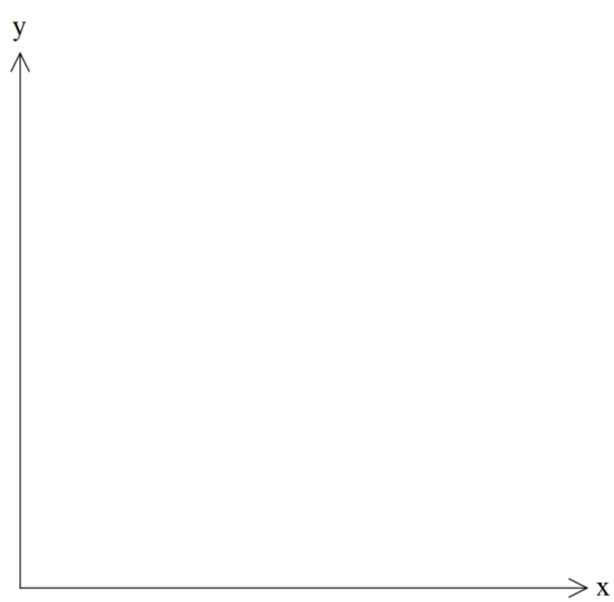
b) Explain what the data is telling us specifically about the correlation between the volume and the mass of the beaker and ethanol.

Assignment 2: Displaying Data and Describing Trends

1. Enter the following information and create a scatterplot on your calculator.

   a) Draw a sketch of the scatterplot on the axes below. (Remember to indicate the scale and use labels.)

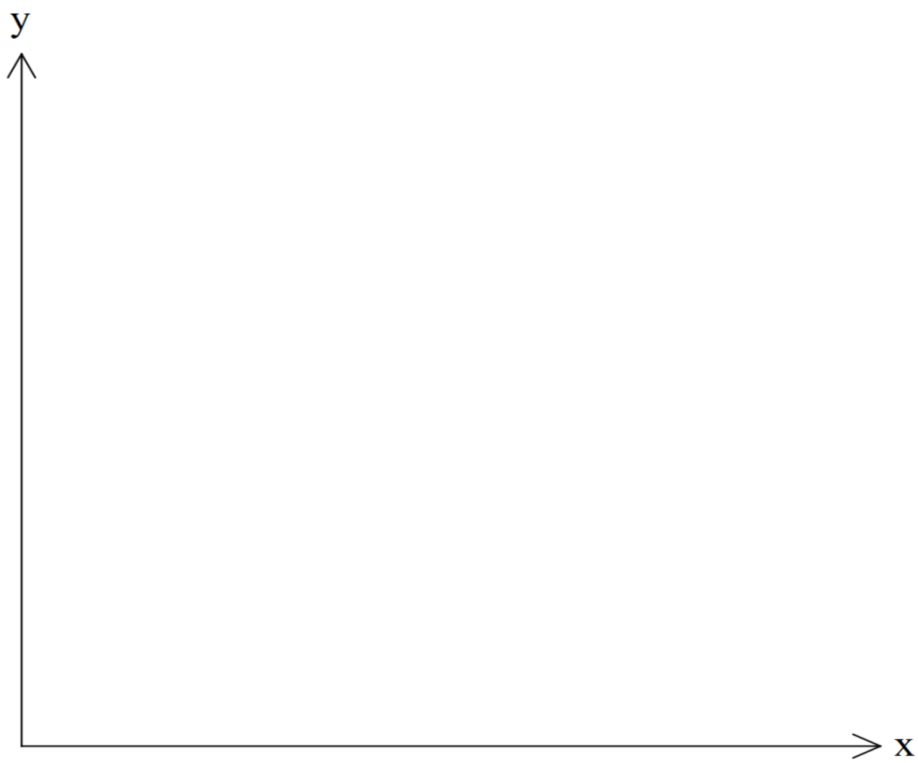
Sandwich	Total Fat (g)	Total Calories
Hamburger	9	260
Cheeseburger	13	320
Quarter Pounder	21	420
Big Mac	31	560
Arch Sandwich Special	31	550
Arch Special with Bacon	34	590



   b) Describe any relationships or trends in the data.

2. The table shows the sales data from an ice cream shop for one week:
   a) Draw a sketch of the scatterplot. (Remember to indicate the scale and context.)

Temperature °C	11.9°C	14.2°C	15.2°C	16.4°C	18.5°C	19.4°C	22.1°C
Ice Cream Sales $	185	215	332	325	406	412	522



   b) Describe any trends or relationships in the data.

   c) What would you predict sales to be if the temperature was 20°C?

3. The following table gives the heights and weights of 10 friends:
   a) Draw a sketch of the scatterplot. (Include the scale and label for each axis).

Name	Height (cm)	Weight (kg)
Albert	180	87
Beth	176	55
Cindy	144	52
David	195	94
Emily	159	87
Frank	185	79
Gary	166	59
Helen	173	64
Ida	149	45
Jeremy	168	77

   b) Describe any trends or relationships in the data.

Lesson 3: Using Technology to Display Data and find a Linear Regression Equation

Goals:
- To develop linear regression equations from a table of data.
- To use regression equations to solve problems.

There are many types of relationships between two quantities. In this unit, we will focus on linear relationships. We can use the graphing calculator to interpret the data and determine an equation that best fit the data. This type of equation is called a regression equation.

Finding equations that match real life situations is called "mathematical modelling". Mathematical models are used in physics, biology, earth science, meteorology, computer science, artificial intelligence, economics, psychology, sociology, political science, and many other fields. Mathematical models help to describe and explain situations and are used to make predictions. When we develop a regression equation, we can use it to predict additional values that aren't on the initial table of values.

Example 1: Linear Regressions

When a scatterplot of data shows a linear pattern, we can develop a linear equation to model the situation, as shown in the following examples.

The table shows some data collected regarding the relationship between the size of a house in a certain neighbourhood and the price of the house.

a) Determine the linear regression equation that models this situation. The instructions on how this is done are found on the next page.

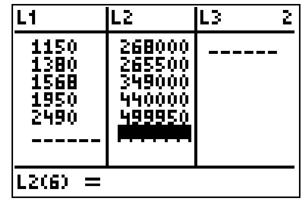
Size (ft^2)	Price ($)
1150	268 000
1380	265 500
1568	349 000
1950	440 000
2490	499 950

b) Use the regression equation to determine the price of a house that is 2000 ft^2.

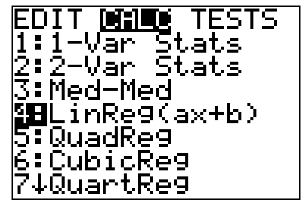
c) Determine the price of a house that is 3400 ft^2.

Finding Regression Equations on the Ti – 84

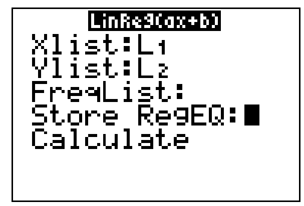
1. Press the STAT button. Choose 1:Edit from the menu and then enter your data (x-values in L_1 and y-values in L_2).



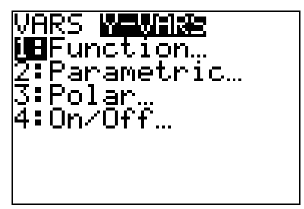
2. After entering your data, press the STAT button (again) and then press the right arrow key to highlight the CALC menu. From that menu (pictured), choose 4:LinReg or 5:QuadReg (depending on function type).



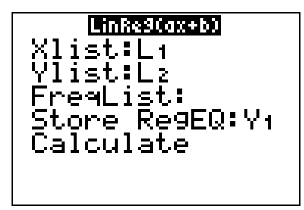
3. Once you have chosen the appropriate function type, use the down arrow to move your cursor to the line that says Store RegEQ (pictured).



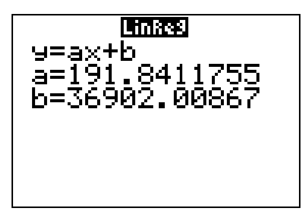
4. Press the VARS button, and then use the right arrow to highlight the Y–VARS menu.



5. Press ENTER twice (once to select option 1:Function…, and again to select option 1:Y_1).



6. Now select the Calculate option at the bottom of the menu. You will be taken to a screen with information about your regression equation – you would substitute the given values into the equation in the proper places.
   For the example at right, you would record the equation as:
   $$y = 191.84x + 36902.01$$



Example 2

Temperatures can be measured in degrees Celsius or degrees Fahrenheit. A temperature of 32ºF is equal to a temperature of 0ºC.

The table shows how some temperatures in Fahrenheit and Celsius compare.

Temperature in Degrees Fahrenheit	Temperature in Degrees Celsius
32	0
212	100
77	25
96.8	36

a) Determine the regression equation for the data.

b) Use the regression equation to determine the temperature in °C if it is 10ºF.

Example 3

A rental company charges a flat fee of $150 and an additional $2.50 per km driven to rent a moving van. The table shows the cost of some rentals.

Number of kms	Cost ($)
0	$150
30	$225
58	$295
90	$375

a) Determine the regression equation for the data.

b) Use the regression equation to determine the cost of the rental if the van was driven 65 km.

c) Use the regression equati0n to determine the cost of the rental if the van was driven 225 km.

Assignment 3: Using Technology to Display Data and find a Regression Equation

1. a) Determine the regression equation for the data shown.

Lake	Area (km^2)	Depth (m)
Superior	82 100	406
Huron	59 600	229
Michigan	57 800	281
Erie	25 700	64
Winnipeg	24 400	18

b) Use the regression equation to predict the depth of a lake that has an area of 43 000 km^2.

2. a) Determine the regression equation for the data shown.

Make of Car	Engine Size (L)	Fuel Use (L/100 km)
Chevy Chevette	1.6	5.7
Chevy Sprint	1.0	4.3
Chrysler 5th Avenue	5.2	10.1
Ford Mustang	5.0	8.5
Honda Civic	1.5	6.2
Jaguar XJ-S	5.3	12.5
Plymouth Colt	1.5	5.8
Pontiac Grand Prix	5.0	9.6

b) Describe any trends or relationships.

c) Use the regression equation to predict the fuel use of a 4 L engine.

3. a) The table shows data relating the height of a football after it is kicked. Determine the regression equation that best represents this data.

Time (s)	Height (m)
0	1.2
.5	10.48
1.7	22.74
2	23.6
3.2	18.22
4	6.8

b) Use the regression equation to determine the height of the football after 2.4 seconds.

c) Determine the height of the ball after 5 seconds.

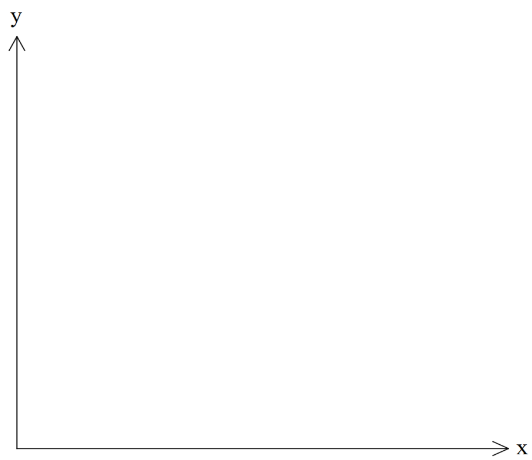
4. A golf instructor kept track of the hours his students practiced. He then compared these hours to the students' golf handicap numbers. The results are recorded below:

Player	Hours of Practice	Golf Handicap
Alice	40	3
Bill	23	12
Charles	55	4
Dina	12	18
Ernie	26	15
Frank	37	15
George	62	3

a) Determine the regression equation for the line of best fit.

b) Use the regression equation to predict the golf handicap for a player that practices for 65 hours.

5. a) Sketch the scatterplot of the data shown in the table



Year	**Car Sales ($1000's)**
1950	325
1955	387
1960	448
1965	709
1970	640
1975	989

b) Describe any relationships between the year and car sales.

c) Determine the regression equation for the data.

d) Use the equation to predict the car sales in the year 2000.

e) What assumption are we making in part d?

Lesson 4: Identifying Characteristics of Linear Functions from a Given Equation

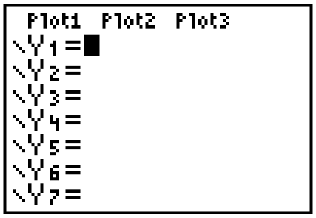
GOAL:

- To identify and describe the characteristics of a linear function including:
  - Leading coefficient
  - Slope
  - End Behaviour
  - y-intercept
  - x-intercept
  - Domain and Range

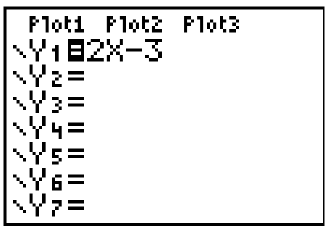
Equations that have straight line graphs are called linear equations. Sometimes these are referred to as *linear functions.* We can determine a lot of information about the graph of the function just by examining the equation.

In the previous lesson, we saw that we could determine an equation if we were given a table of data. We could use that equation to answer further questions about the relationship of the data. We now move on to questions where we are given the *equation.* There is a lot of information we can determine about the relationship of the data just by studying the equation itself as well as from the graph of the equation.

If we are given an equation, we can enter it into the calculator. Start by pressing the $Y =$ key, which gives the following calculator screen:



To graph the straight line $y = 2x - 3$, we make sure the cursor is in the correct position beside $\backslash Y_1 =$ and then we enter the remaining symbols: $2x - 3$.

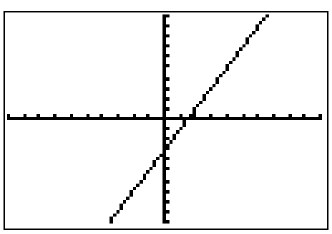


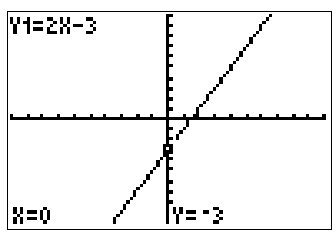
To enter the variable x use the key found to the right of the green ALPHA key.

x, T, θ, n

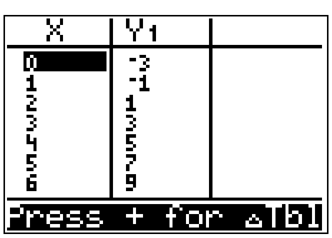
Also, be sure to use the subtraction symbol for minus and not the negative sign.

Press the GRAPH key and you should see the following, where the x-axis and y-axis are visible and the line slants across the screen. Pressing TRACE shows the second screen.

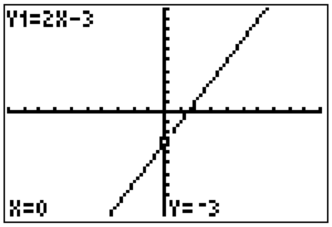




The calculator automatically creates a table of values when an equation is entered. You can view the table by pressing the 2nd key followed by the GRAPH key.



We can describe the characteristics of the function $y = 2x - 3$ by stating that it:



Slope Direction: Rises to the right.

Sign of leading coefficient: Positive

End Behaviour: Q III to Q I.

Domain: $x \in R$

Range: $y \in R$

Domain and Range

Domain: is a list of all the possible x-values of a function. Inspect the side to side growth of a graph to determine its domain.

Range: is a list of all the possible y-values of a function. Inspect the up and down growth of a graph to determine its range.

For linear functions, because the possible values for both x and y are limitless, both the domain and range are the set of all Real Numbers, which we represent by the math symbols:

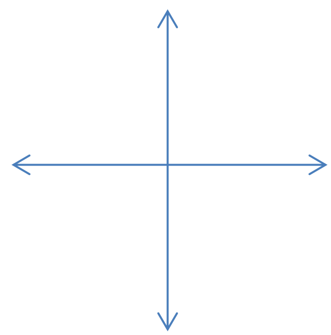
$$x \in R \text{ and } y \in R.$$

However, be aware that when we use linear functions to model 'real world' problems, the domain and range may be restricted to certain values for x and y.

Example 1: Analyzing Linear Functions when we are given the Equation

Enter the equation into your calculator: $y = -3x + 5$

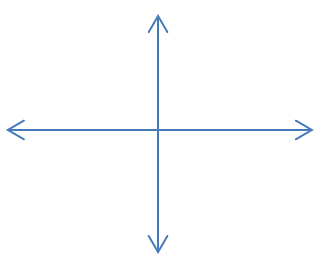
Draw a sketch of the function. Label at least two points on the graph.



Slope Direction	
Sign of Leading Coefficient	
End Behaviour	
Domain	
Range	

Example 2

a) Draw a sketch of the function $y = -4$

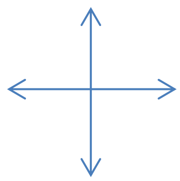
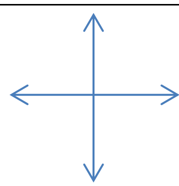
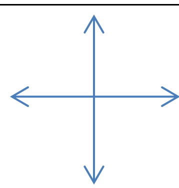
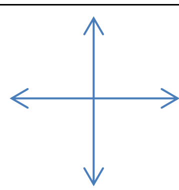
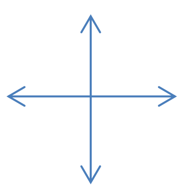
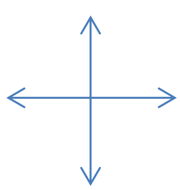


b) Describe the end behavior and the slope.

c) This equation has a leading co-efficient of 0. Explain why.

Example 3: Analyzing Linear Functions

Use the graphing calculator to help you analyze each linear function and complete the table.

Function	Sketch	Sign of Leading Co-efficient	Slope Direction	End Behaviour
$y = 0.5x + 4$				
$y = -2.2x + 12$				
$y = -5$				
$y = -\frac{1}{3}(x - 1)$				
$y = 3(x - 8)$				
$y = 3 - 4x$				

Assignment 4 Identifying Characteristics of Linear Functions from a Given Equation

Analyze each of the following functions.

1. $y = 3.7x + 1.2$

   Sign of Leading Coefficient _____

   Slope direction _______

   End Behaviour ________

2. $y = 4x - 5$

   Sign of Leading Coefficient _______

   Slope direction _______

   End Behaviour _________

3. $y = -2(x + 3.7)$

   Sign of Leading Coefficient _____

   Slope direction _______

   End Behaviour ________

4. $y = -4x + \frac{1}{2}$

   Sign of Leading Coefficient _______

   Slope direction _______

   End Behaviour _________

5. $y = \frac{2}{3}(x + 3)$

   Sign of Leading Coefficient _____

   Slope direction _______

   End Behaviour ________

6. $y = 2 - 0.85x$

   Sign of Leading Coefficient _______

   Slope direction _______

   End Behaviour _________

7. $y = 5$

   Sign of Leading Coefficient _____

   Slope direction _______

   End Behaviour ________

8. a) Which of the equations in # 1-7 are in the form $y = ax + b$?

   b) If a is a positive number what can you conclude about the graph?

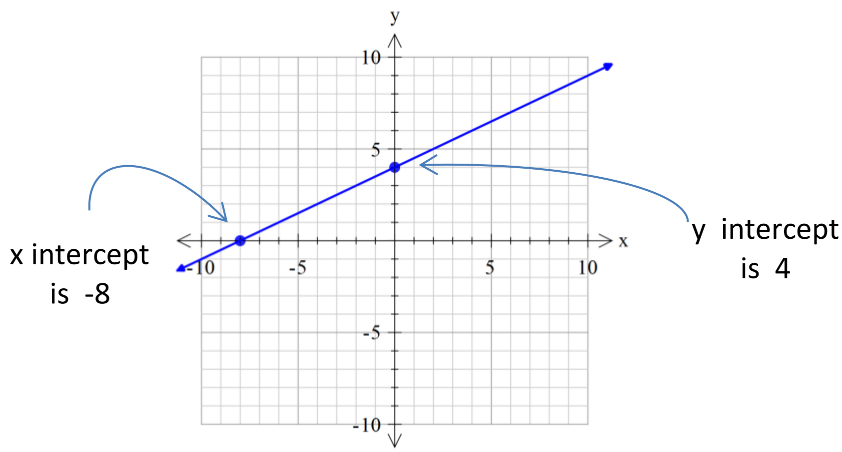
   c) If a is a negative number what can you conclude about the graph?

Lesson 5: Finding the Intercepts of a Linear Function

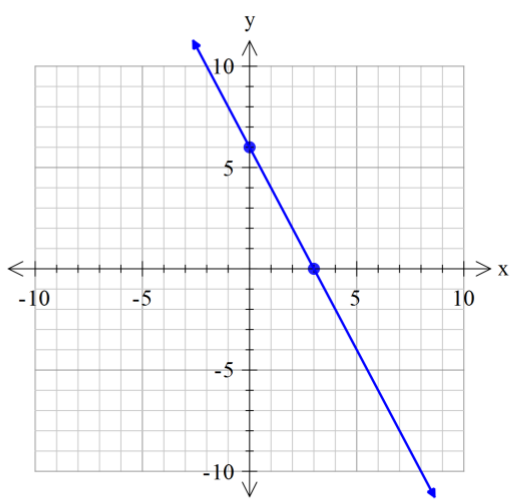
GOALS

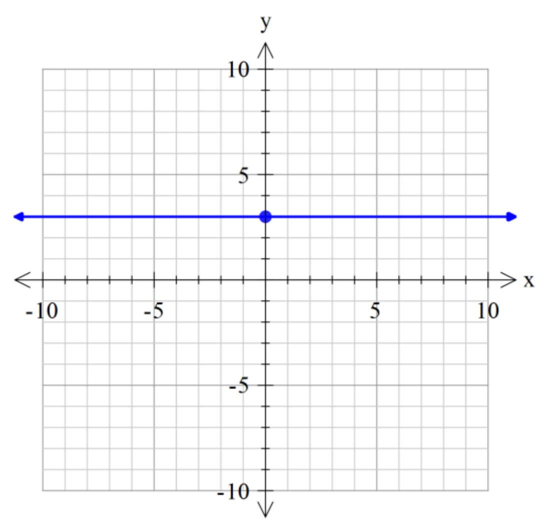
- To determine the x and y intercepts of a linear function.
- To solve problems involving the x and y intercepts of a linear function
- To recognize limitations on the x values and y values in a contextual problem.

The x and y intercepts are two special points on the graph of any function.
The y-intercept is where the equation crosses the y-axis and the$x$-intercept is where the graph crosses the x-axis.



Example 1: **Determining Intercepts**
Label the x and y intercepts on each of the following graphs.

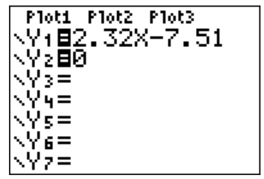
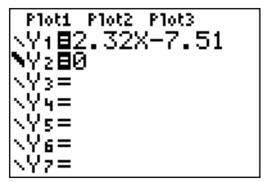
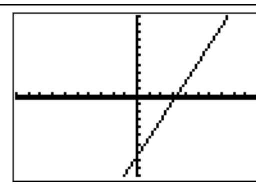
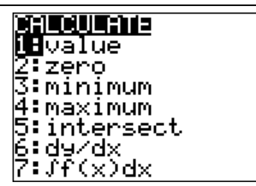
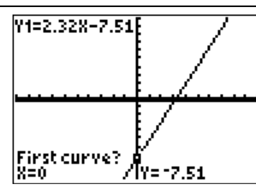


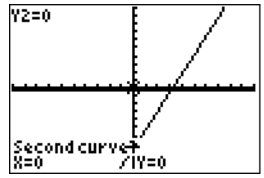
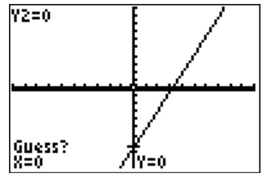
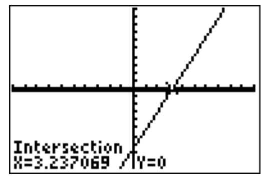


Sometimes your graphing screen will automatically show the y intercept when you press the TRACE key. At other times you may need to enter the $\boldsymbol{x}$ value of 0 (press TRACE, press the 0 key, and press ENTER). This always works because the y intercept always has an x value of 0.

Finding the x intercept is more work.
To find the x intercept we must find the point of intersection between the given equation and the x axis. To find the x-intercept you need to make your *y-value* 0. This works because the x-intercept always has a y-value of 0.

The following screen shots and comments show how to find the x intercept for the function $y = 2.32x - 7.51$.

Instructions	**What you should see**
Press the Y= button. Enter the given equation in Y_1. Enter $Y_2 = 0$ on the next line. (y = 0 is the equation of the x axis.)	
Use the arrow keys to move the cursor onto the slash symbol in front of Y_2. Press ENTER The slash symbol should darken and blink. This step is optional, but makes it easier to see the x axis.	
Now Press GRAPH. Notice that the x axis is a little darker than usual.	
At this point we are ready to use the calculator to find the intersection point of the function and the x axis.	
Press the 2nd button then press the TRACE button to access the CALCULATE menu.	
Choose 5: intersect Notice that the calculator is now asking you a question (near the bottom of the screen): “First Curve?”	
Instructions continue on next page…	

Press ENTER. This is like saying "yes". Notice that you now get a new question: "Second Curve?"	
Press ENTER again. (To say "Yes" again.) You will see one last question: "Guess?"	
Press ENTER again. You should now see the point of intersection between the graph and the x-axis. This is your x-intercept.	
When recording your x-intercept on a test or assignment, note that you only need to provide the x-value: You would write $\boldsymbol{x = 3.24}$.	

Example 2: Determining Intercepts using the Graphing Calculator

a) Find the x and y intercepts for:

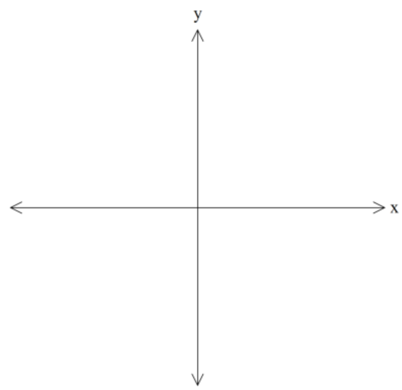
$$y = -5.2x + 0.87$$

b) Find the x and y intercepts for

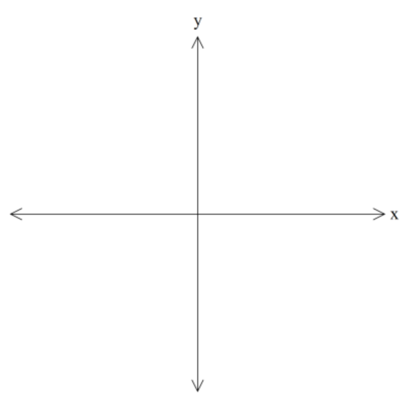
$$y = 3.45x + 7.5$$

Example 3: Graphing Linear Functions using Intercepts

a) Draw the graph of $y = 2.5x - 14$. Label both intercepts.



b) Draw the graph of $y = -2.5x - 14$. Label both intercepts.



Assignment 5: Finding the Intercepts of a Linear Function

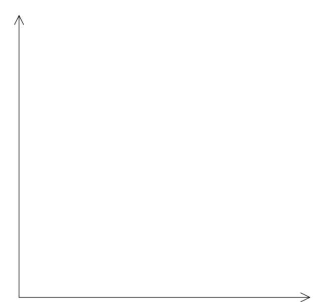
1. Sketch the graph of each of the following functions. Clearly label all intercepts.
   a) $y = 3.41x + 1.22$
   b) $y = 4.33x - 5$
   c) $y = -(x - 5.12)$
   d) $y = -4x + \frac{1}{2}$
   e) $y = -\frac{7}{2}$
   f) $y = 2 - 9.5x$

2. State the requested characteristics of the function $y = 6.28x + 1.57$.
   a) Sign of Leading Coefficient
   b) Slope direction
   c) Value of the y intercept
   d) Value of the x intercept
   e) End Behaviour
   f) Domain and Range

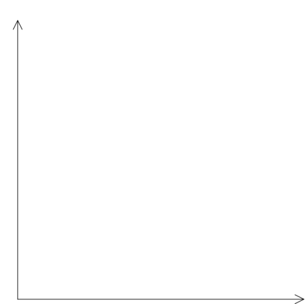
3. A hot air balloon is flying at an altitude of 1236 m. It begins descending at a rate of 10 m per minute. The balloon's height, from the moment it begins its descent, until it reaches the ground, is given by the function:

$$H = -10t + 1236$$

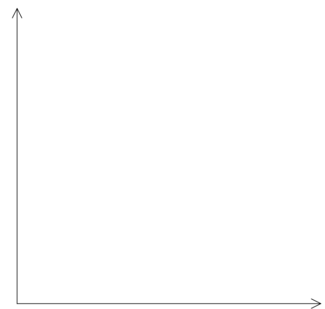
   where t represents the time, in minutes since the descent began, and H represents the height of the balloon above the ground, in metres.
   a) Sketch the graph of this function.
   b) What does t represent?
   c) What does H represent?
   d) What does the H intercept represent?
   e) What does the t-intercept represent?
   f) How long does it take the balloon to reach the ground?
   g) Does the graph continue in both directions? Explain.
   h) What is an appropriate range for this problem?



4. Amil currently has \$200 and spends \$20 each week. The function $A = 200 - 20w$ describes his spending pattern.
   a) Draw the graph of the function.
   b) What does the variable w represent in this situation?
   c) What does the variable A represent?
   d) What does the A-intercept represent?
   e) What does the w-intercept represent?
   f) Determine the value of w when $A = 0$.
   Explain the meaning of this value of w.



5. The cost to rent a hall is given by the equation $\boldsymbol{C = 100 + 5n}$, where $\boldsymbol{n}$ represents the number of party guests and $\boldsymbol{C}$ represents the cost.
   a) Draw the graph of this function.
   b) What does the C-intercept represent?
   c) What does the n-intercept represent?
   d) Does the graph continue in both directions? Explain.
   e) What is an appropriate range for this situation?



6. Weight on the moon is not the same as it is on Earth because of differences in the force of gravity. The function $M = \frac{1}{6}E$ can be used to approximate your weight M on the moon, where E represents your weight on Earth.
   a) Does the function indicate that you would be heavier or lighter on the moon than on the Earth? Explain.
   b) If a person weighs 80 kg on Earth, how much would the person weigh on the moon?
   c) What is an appropriate domain for this problem?

Lesson 6: Applications of Linear Functions

GOAL:
- To solve contextual problems modelled by linear functions.
- To solve contextual problems involving the characteristics of quadratic functions.

This lesson incorporates all of the skills you've learned so far in this unit.

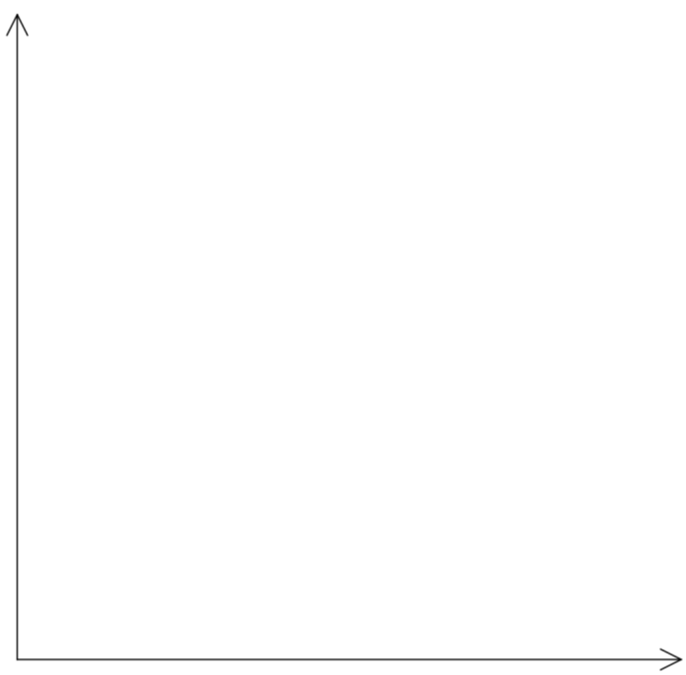
Example 1: Linear Application with Equation Given

Maya is making necklaces for a craft fair. It costs her \$2.50 to make each necklace, and she needs to pay \$49 to rent a table at the craft fair. This situation is modelled by the following equation:

$$\boldsymbol{C = 2.5n + 49}$$

where $\boldsymbol{n}$ represents the number of necklaces made and $\boldsymbol{C}$ represents the total cost.

a) Sketch a graph of this function. Remember to include labels on your axes and appropriate scales.



b) Find Maya's total cost if she plans to make 200 necklaces.
Note: You can substitute the value for $\boldsymbol{n}$ (200) into the equation and solve algebraically or you can use the same technique you learned in Lesson 5 for finding the $\boldsymbol{y}$ intercept of a function.

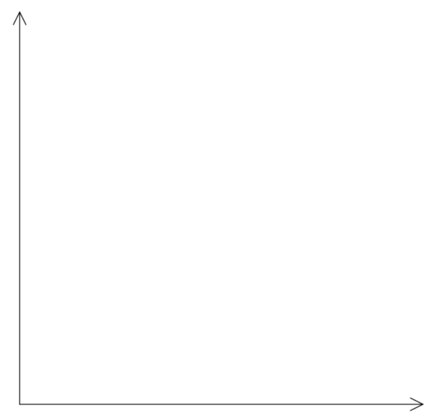
Example 2: Linear Application with Equation Given

Ahmed starts the week with \$165 and spends \$8.50 per day. This situation is modelled by the following equation:

$$L = 165 - 8.5d$$

where d is the number of days that have passed and L is the amount of money he has left.

a) Sketch the graph.



b) State the domain and range of this situation.

c) How much money does Ahmed have after 5 days have passed?

d) How much money does Ahmed have after 30 days have passed?

e) When will Ahmed have \$100 left? This is the same as asking you to find the x value when you know the y value. In this case, you will be finding the d-value when you know the L-value. This is the same process that you used to find the x-intercepts of functions. You will enter an equation into the $y =$ screen ($Y_2 = 100$) and then use 2nd CALC: 5: intersect to solve for the d-value. Your teacher will guide you through the process.

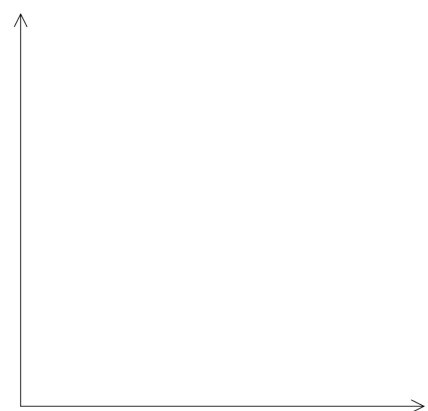
Example 3: Linear Application with Table of Values Given

The cost of tacos purchased depends on the number ordered. The table below shows the cost of purchasing tacos from a food truck.

Number of tacos ordered	1	2	3	5	8
Cost $	3.50	7.00	10.50	17.50	28.00

a) Determine the linear equation that models this situation.

b) Sketch the graph of this situation



b) How much will it cost if 20 tacos are ordered?

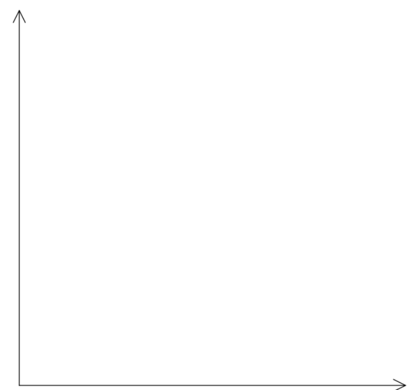
c) How many tacos could you buy with $100.00?

Example 4: Linear Application with Verbal Description Given

The student council is planning to hold a fundraising dinner. They will profit \$8 per dinner ticket and the other costs for the dinner are for the hall rental and the music, which total to \$300. Fill in the table below for this situation.

# of Tickets Sold	0	15	25	50	100
Total Profit \$	−300				

a) Graph the scatterplot of the data.



c) Is the relation linear? Explain. If so, sketch the linear regression equation on the grid above, and write the regression equation below.

d) How many tickets do they have to sell in order to ‘break even’? To break even means to have a ‘profit’ of \$0.

e) What is the profit if 180 tickets are sold?

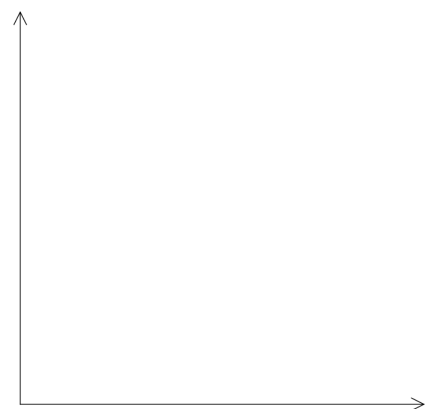
f) If their goal is to make \$1000, how many tickets do they have to sell?

Example 5: Linear Application with Verbal Description Given

There is 8 cm of snow on the ground. It is melting at a rate of 0.5 cm per hour. Complete the table shown below.

Time (hours)					
Amount of Snow (cm)					

a) Graph the scatterplot of the data.



b) Is the relation linear? Explain. If so, sketch the linear regression equation on the grid above, and write the regression equation below.

c) How will it take for the snow to melt completely?

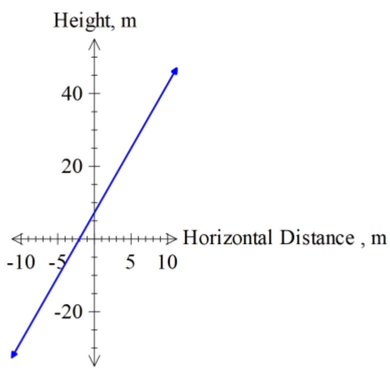
d) When will the snow melt to a depth of 3.75 cm?

e) What are we assuming regarding our answers to parts (c) and (d)?

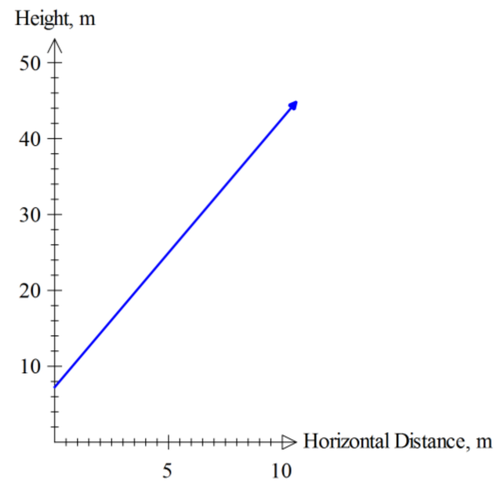
Example 6: Using Appropriate Graphs for Applications of Linear Functions.

A rocket follows a path given by $y = 3.54x + 7.25$, where x represents the distance from the launch pad and y represents the height. Two graphs are drawn.

Both graphs show the equation accurately but the second graph fits the situation better. Explain why.



Graph 1



Graph 2

Note that the domains and ranges of these two graphs are different although they use the same linear function to model both situations.

Graph 1: Domain: $x \in R$

Range: $y \in R$

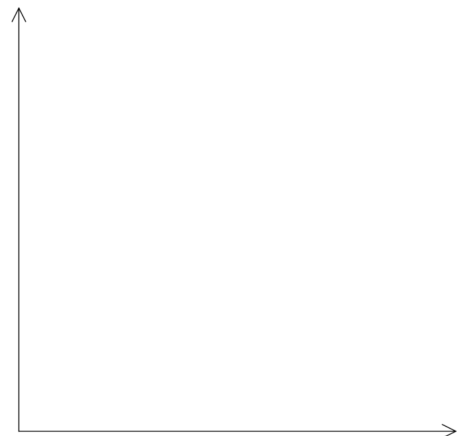
Graph 2: Domain: $x \geq 0$

Range: $y \geq 7.25$

The domain and range of Graph 2 is restricted because the rocket is only moving forward from its starting point and it is launched from a height of 7.25 m and the height only increases from that value.

Example 5: Applications of Linear Functions

A ball moves from a rooftop to the ground, according to the equation $V = -3.5h + 12.5$ where h is the horizontal distance, in metres, and V is the vertical distance, in metres. Draw the graph of this situation.



a) What does h represent?

b) What does V represent?

c) What does the V-intercept represent?

d) What does the h-intercept represent?

e) How far from the base of the building did the ball land?

f) Does the graph continue forever in both directions? Explain.

g) What are the limitations on the values of V?

h) What are the limitations on the values of h?

Assignment 6: Applications of Linear Functions

1. The cost of sending a school volleyball team to a tournament follows the equation $y = 21x + 140$, where x is number of players and y is the cost, in dollars.
   a) Sketch a graph of this situation. Be sure to include labels and scale.
   b) How much will it cost to send 20 players?
   c) How many players can the school afford to send if the budget is $980.00?

2. Carlos has $1000 set aside to give to his three children. He will give each of them $15 a week until the money runs out. The equation that represents this situation is $y = -45x + 1000$, where x is time (in weeks) and y is money left over, in dollars.
   a) Sketch the graph of this situation.
   b) How much money will Carlos have left after 2 weeks?
   c) How many weeks will it take for the money to drop to nothing? Round to the nearest week.

3. The cost of a wedding dinner varies with the number of guests who attend, as shown in the table below.

Number of Guests	50	80	100	120	200
Cost	2150	3050	3650	4250	6650

   a) Determine the equation that best models this data.
   b) How much will it cost for 550 guests to attend?
   c) If the couple has a total of $12 500 budgeted for the wedding and $2500 of this is for other expenses, how many guests can they invite to the dinner?

4. A car is travelling according to the equation

$$d = 90h + 55$$

   where h represents the number of hours that have passed and d represents the distance from Winnipeg, in kilometres.
   a) Sketch the graph. Give the coordinates of at least 2 points.
   b) Determine the distance from Winnipeg after 3.5 hours have passed.
   c) Determine the distance from Winnipeg after 30 minutes have passed.

5. A giant bison is an extinct animal. Scientists use fossil bones to estimate the size of these animals. The equation

$$L = 2.4b - 7.9$$

is used to model the relationship between the length of the humerus bone, b, in centimetres, and a bison's front limb length, L, in centimetres.

a) Sketch a graph of the equation.

b) Determine the front limb length, to the nearest cm for each bison:
   - An extinct giant bison with a fossil humerus bone length of 40.2 cm
   - A modern North American bison with a humerus bone length of 32.6 cm

c) By what percent was the giant bison taller than the modern bison?

6. Soo-Young is driving to Calgary. Her distance from Calgary over the number of hours that she has been driving is shown in the table below.

Time (hours)	0	2	3	5	7
Distance (km)	1200	984	876	660	444

a) Sketch the scatterplot formed by these points.

b) Determine the equation that best represents this data.

c) How far is Soo-Young from Calgary after driving for 7 hours?

d) How long has Soo-Young been driving if she is a distance of 175 km from Calgary?

e) How far is Soo-Young from Calgary after driving for 20 hours? Why is this answer strange?

7. Maria just purchased a new compact car, hoping to save money driving to work and back. Maria tracked her expenses for six months in order to determine the average monthly cost of operating her new car. In addition to the cost of gas, which is $1.20 per litres, she also calculated a monthly cost of $75 per month, which includes maintenance and insurance costs.

a) Fill in the table below based on Maria's cost.

Month	Number of litres purchased	Total Cost of Gas	Total Monthly Cost
1	60		
2	70		
3	65		
4	50		
5	45		

b) Determine the equation that models the total monthly cost as a function of the number of litres of gas purchased.

c) Use the equation to determine the total monthly cost if Maria purchased 90 litres of gas.

d) Determine the number of litres of gas purchased if Maria's total monthly cost was $105.

8. The cost of a cab ride follows the equation $1.71x + 3.50 = y$, where x represents the number of kilometres travelled and y is the cost.
   a) How much will it cost for a 5 km cab ride?
   b) Joseph can afford $20.00 for the cab ride. How far can he travel?
   c) Joseph wants to visit his cousins who live 12 km away. He decides to spend $20 for the cab ride and then walk the rest of the way. How far will he have to walk?

9. A certain pizza restaurant charges $12.50 for a basic pizza and $1.25 for each extra topping. Determine an equation for the cost of a pizza as a function of how many toppings are on it. You can use the table below to create some data that models this situation.

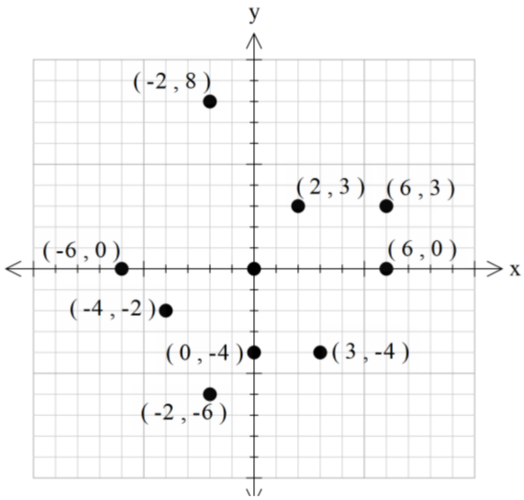
Number of Toppings	0				
Cost of Pizza					

   a) Sketch the scatterplot that represents this data.
   b) Determine an equation that best models this situation.
   c) How much does a 3 topping pizza cost?
   d) Joanie has only $20.00. What is the maximum number of toppings she can afford for her pizza?

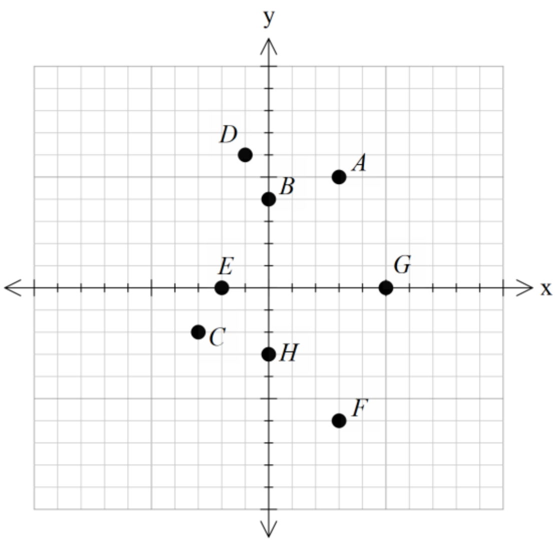
Answer Key

In-Class Activity: Plotting Points Answers

1.

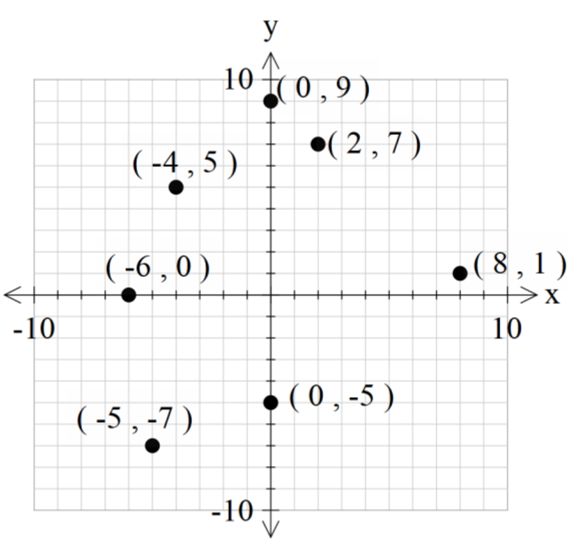


2.



3. a) Q IV b) Q III c) QI d) Q II e) Q III f) Q I g) Q II h) n/a i) Q III j) Q IV

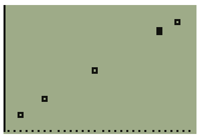
4.

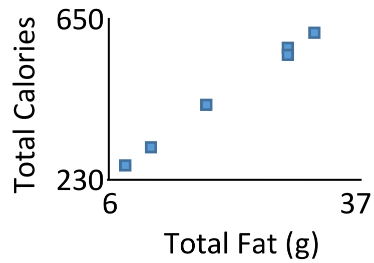


Assignment 2: Answers

It is important to note that your screenshot may not match the answer key exactly, but you should be able to see the same number of points (with the same relationship between the points) on your scattershot.

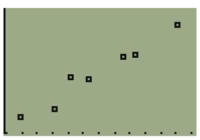
1. a) Screenshot: Sketch:

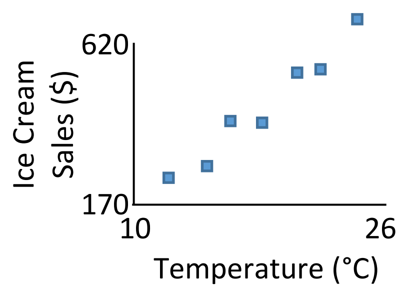




b) The correlation is positive. As the fat content increases, the total calories increase.

2. a) Screenshot: Sketch:

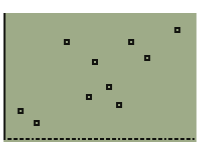


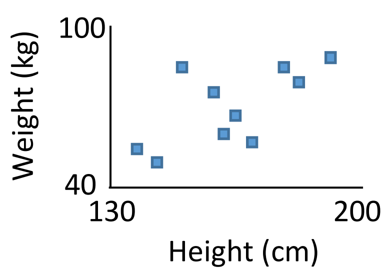


b) The correlation is positive. As the temperature increases, the ice cream sales increase.

c) We would expect the sales to be somewhere between $412 and $522.

3. a) Screenshot: Sketch:



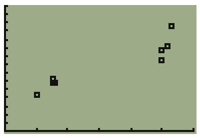


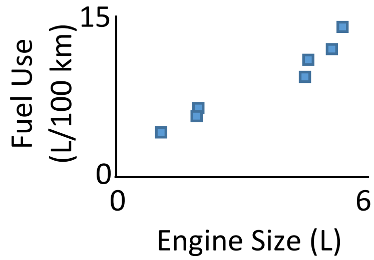
b) The relationship between the two variables may be slightly positive or it may be random.

Assignment 3: Answers

1. a) $y = 0.0064x - 118.65$ b) $155.48\ m^2$

2. a) $y = 1.29x + 3.63$
   b) Screenshot: Sketch:





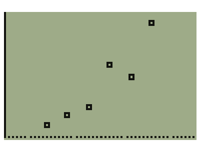
   c) The smaller engines use less fuel and the larger engines use more fuel. There is a positive correlation between engine size and fuel use.

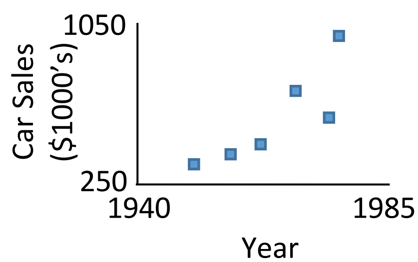
   d) $8.79\ L/100km.$

3. a) $y = -4.90x^2 + 20.98x + 1.20$ b) 23.36 m
   c) The ball would have landed on the ground before 5 seconds.

4. a) $y = -0.31x + 21.27$ b) 1.16; gets rounded to 1.

5. a) Screenshot: Sketch:





   b) There is a positive correlation between the year and car sales; as the years go on, the car sales increase.
   c) $y = 24.8x + 273$ OR $y = 24.8x - 48087$, depending on whether you used the raw data as is for Car Sales, or if you multiplied each number in that column by 1000 (since it is in 1000's of dollars.
   c) When x = 2000, y=1513, therefore the model predicts sales would be $ 1 513 000.
   d) We are assuming that the trend continues and that sales will continue increasing.

Assignment 4: Answers

1. Leading co-efficient is Positive
   Slope direction is up
   End Behaviour Q III to Q I

2. Leading co-efficient is Positive
   Slope direction is up
   End Behaviour Q III to Q I

3. Leading co-efficient is Negative
   Slope direction is down
   End Behaviour Q II to Q IV

4. Leading co-efficient is Negative
   Slope direction is down
   End Behaviour Q II to Q IV

5. Leading co-efficient is Positive
   Slope direction is up
   End Behaviour Q III to Q I

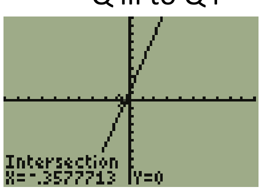
6. Leading co-efficient is Negative
   Slope direction is down
   End Behaviour Q II to Q IV

7. Leading co-efficient does not exist Slope direction is flat or horizontal
   End Behaviour Q II to Q I

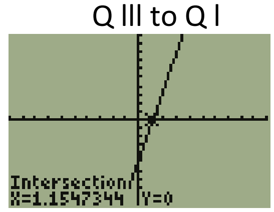
8. a) Question #1, #2, and #4 have equations that are in the form $y = ax + b$.
   b) If ***a*** is positive, then we know that the line slopes upward and the end behaviour is from Q lll to Q l.
   If ***a*** is negative then we know the line slopes downward and the end behaviour is from Q ll to Q lV.

Assignment 5: Answers

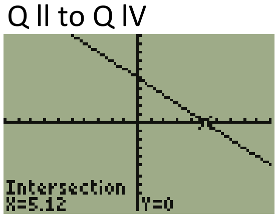
1. a) y intercept is 1.22
   x intercept is – 0.36
   Q lll to Q l



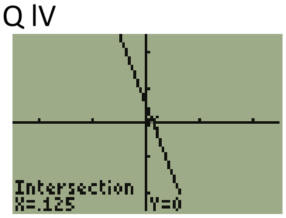
b) y intercept is -5
   x intercept is 1.15
   Q lll to Q l



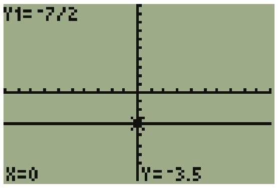
c) y intercept is 5.12
   x intercept is 5.12
   Q ll to Q lV



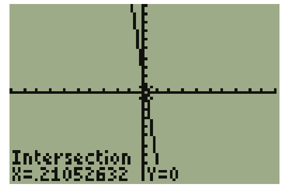
d) y intercept is 0.5
   x intercept is 0.125
   Q ll to Q lV



e) y intercept is – 3.5
There is no x intercept.
Q lll to Q lV



f) y intercept is 2
x intercept is 0.21
Q ll to Q lV



2. Leading co-efficient is positive y intercept is 1.57 x intercept is – 0.25
Slope direction is positive End Behaviour is from Qlll to Q l
Domain: $x \in R$ Range: $y \in R$

3. a) needs graph
b) time in minutes c) Height in metres d) Initial height at time zero
e) time, in minutes, when the balloon reaches the ground f) 123.6 minutes
g) Only positive values make sense for this situation. h) $[0,1236]$

4. a) needs graph
b) w represents the number of weeks c) A represents the amount of money
d) the initial amount of money e) The w intercept represents the time, in weeks , when the amount of money reaches zero. f) When A = 0, w =10. This means that Amil runs out of money after 10 weeks.

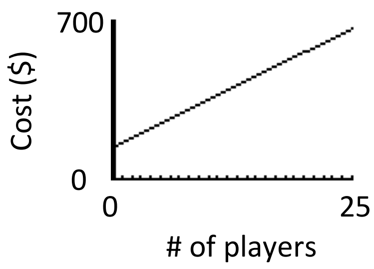
5. a) needs graph
b) n represents the number of guests.
c) C represents the cost.
d) The initial cost which is the cost for renting the hall with no guests.
e) n intercept has no meaning in this problem
f) No. Only positive values make sense in this situation.
g) Since H represents the height in metres, a reasonable set of H values would be from 0 to 1236.

6. a) The function indicates that you will be lighter on the moon, because you divide your Earth weight by 6, to get a smaller weight.
b) 13.33 kg
c) Weight starts at zero and goes up from there, so a reasonable x minimum for this function would be 0. Any positive value would be ok for the maximum. It would also be correct to say that this function has no maximum—you could use a number as big as you like for “weight”.

Assignment 6: Answers

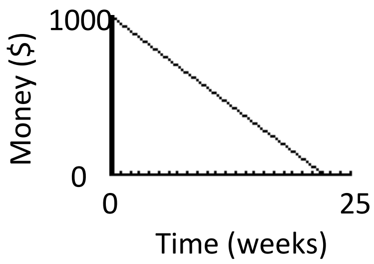
Graph window settings may vary from student to student. Graphs shown with POSSIBLE (appropriate) window settings.

1. a)



b) $560.00 c) 40 players

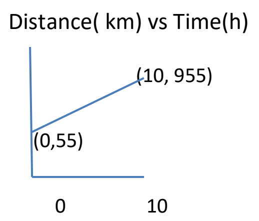
2. a)



b) $910.00 b) 22 weeks

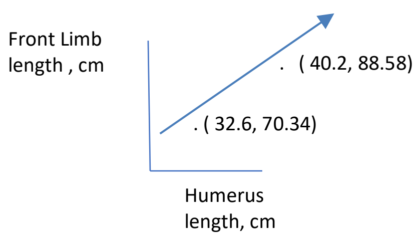
3. a) $y = 30x + 650$ b) $17 150.00 c) 311 guests

4. a)



b) 370 km c) 100 km

5.

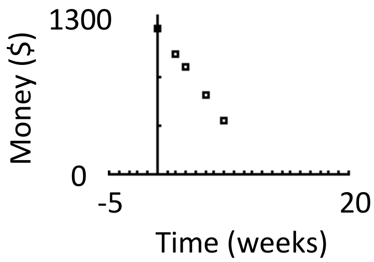


b) Giant bison 89 cm, North American bison 70 cm

c) $\frac{89}{70} = 1.27$ The giant bison was 27% taller than the modern bison.

Note: Using $\frac{88.58}{70.34} = 1.26$, we would say the giant bison was 26 % taller.

6. a)

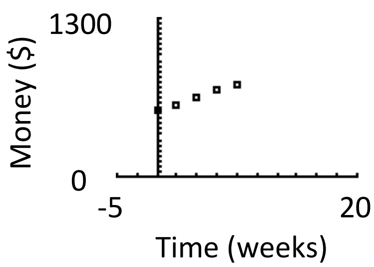


b) $y = -108x + 1200$ c) 444 km d) 9.49 hours
e) –960 km, you are past Calgary

7. a) $y = 1.20x + 75$ b) $183.00 c) 25 liters

8. a) $12.05 b) 9.65 km c) 2.35 km

9. a)



b) $y = 1.25x + 12.50$ c) $16.25 d) 6 toppings

Linear Functions Outcomes Summary

These are the outcomes that have been covered in this unit. **Check off** each box if you are confident that you can demonstrate that skill:

- ☐ I can describe the characteristics of linear function, including end behaviour and y intercept and x intercept(s), domain and range, maximum value, minimum value.
- ☐ I can sketch the graph of a linear function, including the y intercept, x intercepts(s) and one other point or the vertex.
- ☐ I can determine the value of the leading co-efficient and the direction (positive or negative) the function.
- ☐ Given an x value of a linear function, I can find the y value.
- ☐ I can determine the x intercept(s) and the y intercept of a linear function.
- ☐ I can draw a clearly labelled graph of a linear function including all intercepts.
- ☐ I can draw a clearly labelled graph for a contextual problem involving a linear function.
- ☐ I can solve a contextual problem given the graph of a linear function.
- ☐ I can solve a contextual problem given the equation of a linear function.
- ☐ I can solve a contextual problem given a data set or table of values for a linear function.
- ☐ I can determine whether a function is linear.
- ☐ I can solve a contextual problem given a written/verbal description of a linear function.

Ongoing Self-Assessment for Mathematics Students

Understanding
How confident are you in your ability to demonstrate understanding of the outcomes of this unit?

My ability to demonstrate understanding is a: ***STRENGTH*** ☐ ***CHALLENGE*** ☐

Attendance
Did you have consistently good attendance during this unit?

My attendance is a: ***STRENGTH*** ☐ ***CHALLENGE*** ☐

Out of Class Practice
Did you feel that when you needed to practice a math skill outside of class, you were able to do so?

My ability to practice outside of class time is a: ***STRENGTH*** ☐ ***CHALLENGE*** ☐

Accessing Help
If you answered ***CHALLENGE*** to any of the questions above, consider the following options for accessing help in order to be more successful in this course:

- Talk to your ***TEACHER***.
- Make time to visit the ***RESOURCE ROOM (ROOM 104).***
- Get help / support / materials from a ***CLASSMATE.***
- Use any resources provided on a ***CLASS BLOG*** (if available).

You have completed a unit in this Math course. Please take some time to reflect on your thoughts regarding your academic strengths and challenges as they relate to the outcomes of this unit. You can also reflect on any previous outcomes of this course.

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