Name: ______________________ Date: ______________________

6.2 Characteristics of the Equations of Polynomial Functions

YOU WILL NEED
- graphing technology

Keep in Mind

- When a polynomial function is in standard form
  - The maximum number of x-intercepts the graph may have is equal to the degree of the function.
  - The maximum number of turning points the graph may have is equal to one less than the degree of the function.
  - The degree and leading coefficient indicate the end behaviour of the graph of the function.
  - The y-intercept of the graph is equal to the constant term of the function.
- Linear and cubic polynomial functions have similar end behaviour.
  - Negative leading coefficient: the graph extends from Quadrant II to Quadrant IV.
  - Positive leading coefficient: the graph extends from Quadrant III to Quadrant I.
- Quadratic polynomial functions have a different end behaviour.
  - Negative leading coefficient: the graph extends from Quadrant III to Quadrant IV.
  - Positive leading coefficient: the graph extends from Quadrant II to Quadrant I.

TIP

The standard form for a linear function is
$f(x) = ax + b$
where $a \neq 0$.
The standard form for a quadratic function is
$f(x) = ax^2 + bx + c$
where $a \neq 0$.
The standard form for a cubic function is
$f(x) = ax^3 + bx^2 + cx + d$
where $a \neq 0$.

Example 1

Determine the following characteristics of each function, using its equation.

a) $f(x) = 4x + 2$ b) $f(x) = -5x^2 + 2x - 1$

TIP

In your descriptions of characteristics of a function include
- number of x-intercepts
- y-intercept
- end behaviour
- domain
- range
- number of possible turning points

Solution

a) I considered each characteristic of the equation $f(x) = 4x + 2$.

- The value of the greatest exponent is 1, so the degree is 1. Since the degree is 1, the function is linear, so its graph is a line, and the graph has one x-intercept.
- The constant term is 2, so the y-intercept is 2.
- The leading coefficient, 4, is positive, so the graph extends from Quadrant III to Quadrant I.

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- There are no restrictions on x. The domain is $\{x \mid x \in R\}$.
- There are no restrictions on y. The range is $\{y \mid y \in R\}$.
- This function is linear, so it has no turning points.

b) I considered each characteristic of the equation $f(x) = -5x^2 + 2x - 1$.

- The value of the greatest exponent is 2, so the degree is 2. Since the degree is 2, the function is quadratic, so its graph is a parabola, and the graph may have 0, 1, or 2 x-intercepts.
- The constant term is -1, so the y-intercept is -1.
- The leading coefficient, -5, is negative, and the equation is quadratic, so the graph extends from Quadrant III to Quadrant IV.
- There are no restrictions on x. The domain is $\{x \mid x \in R\}$.
- The range is $\{y \mid y \leq \text{maximum}, y \in R\}$.
- This function is quadratic, so it has one turning point.

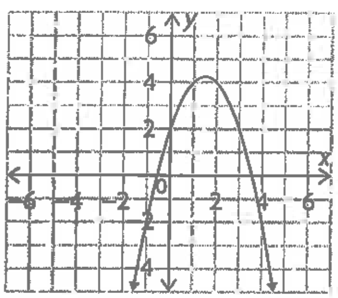
TIP

Quadratic functions always have one turning point. Cubic functions may have two turning points, or none.

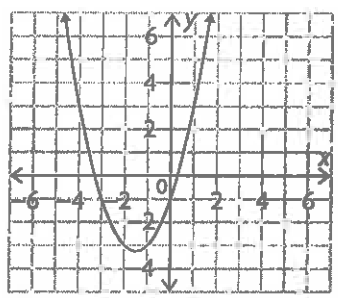
Example 2

Match each graph to the correct polynomial function.

A.



B.



i) $f(x) = x^2 + 3x - 1$ **ii)** $g(x) = -x^2 + 3x + 2$

Solution

Step 1. I looked at the number of turning points in each graph.

Each graph has one turning point, so both $f(x)$ and $g(x)$ are quadratic functions.

Step 2. I looked at the end behaviour of each graph.

Graph A extends from Quadrant III to Quadrant IV, so the leading coefficient must be negative. Graph A matches with $g(x)$.

Graph B extends from Quadrant II to Quadrant I, so the leading coefficient must be positive. Graph B matches with $f(x)$.

Step 3. I verified my conclusion by looking at the y-intercepts.

The y-intercept of Graph A is 2. The constant term of $g(x)$ is 2, so again, the graph and equation match.

The y-intercept of Graph B is -1, matching the constant term of $f(x)$.

Graph A matches with $g(x)$, and Graph B matches with $f(x)$.

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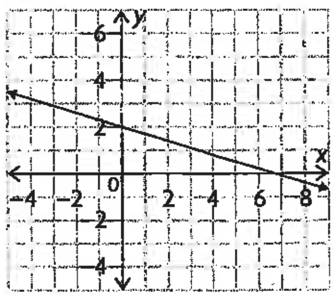
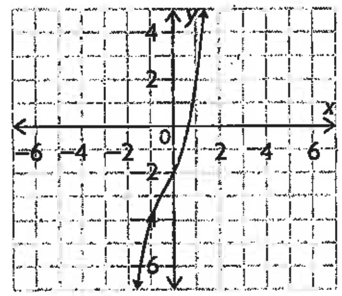
Practice

1. Match each graph with the correct polynomial function. Provide your reasoning.

A.



C.



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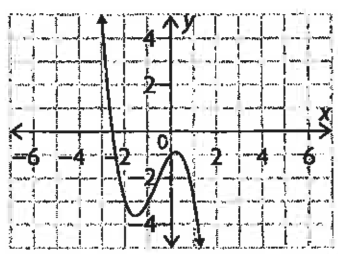
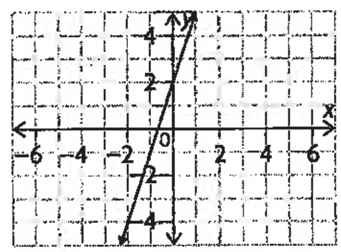


D.



i) $f(x) = -x^3 - 2x^2 + x - 1$

ii) $g(x) = 3x + 2$

iii) $h(x) = -0.3x + 2$

iv) $j(x) = x^3 + x^2 + 2x - 2$

Graph A

number of x-intercepts: ___

y-intercept: ___

end behaviour: from Quadrant ___ to Quadrant ___, so the leading coefficient is ____________

domain: ____________ range: ____________

number of turning points: ___

Graph A represents a ____________ polynomial function. It matches with function ___.

Graph B

number of x-intercepts: ___

y-intercept: ___

end behaviour: from Quadrant ___ to Quadrant ___, so the leading coefficient is ____________

domain: ____________ range: ____________

number of turning points: ___

Graph B represents a ____________ polynomial function. It matches with function ___.

Graph C

number of x-intercepts: ___

y-intercept: ___

end behaviour: from Quadrant ___ to Quadrant ___, so the leading coefficient is ____________

domain: ____________ range: ____________

number of turning points: ___

Graph C represents a ____________ polynomial function. It matches with function ___.

Graph D

number of x-intercepts: ___

y-intercept: ___

end behaviour: from Quadrant ___ to Quadrant ___, so the leading coefficient is ____________

domain: ____________ range: ____________

number of turning points: ___

Graph D represents a ____________ polynomial function. It matches with function ___.

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2. Write a polynomial function that satisfies each set of characteristics.

a) extending from Quadrant III to Quadrant IV, one turning point, y-intercept of 4

b) degree 1, decreasing, y-intercept of -2

c) extending from Quadrant III to Quadrant I, y-intercept of -3

d) two turning points, y-intercept of 5

NUMERICAL RESPONSE

3. State the characteristics of each polynomial function.

a) $f(x) = -3x^2 - 2x + 1$

- degree: ___
- leading coefficient: ___
- constant term: ___
- number of x-intercepts: ___________
- y-intercept: ___
- extends from Quadrant ___ to Quadrant ___
- domain: ___________
- range: ___________
- number of turning points: ___

b) $h(x) = 4x^3 + 2x^2 - x + 34$

- degree: ___
- leading coefficient: ___
- constant term: ___
- number of x-intercepts: ___________
- y-intercept: ___
- extends from Quadrant ___ to Quadrant ___
- domain: ___________
- range: ___________
- number of turning points: ___

WRITTEN RESPONSE

4. The life expectancy of Canadian males born from 1920 to 2008 can be modelled by the polynomial function $E(y) = 0.2339x - 390.8$, where E is the life expectancy in years and x is the year of birth.

a) Describe the characteristics of the graph of the polynomial function. Explain your answer.

b) Would you use this graph to estimate the life expectancy of a male born in the year 1000? Explain.

TIP
In your descriptions of characteristics of a function include
- number of x-intercepts
- y-intercept
- end behaviour
- domain
- range
- number of possible turning points