CHAPTER 3: Transformations of Functions Examples Booklet

40S Pre-Calculus

Winnipeg Adult Education Centre
January 2019

Lesson 1: Horizontal and Vertical Translations

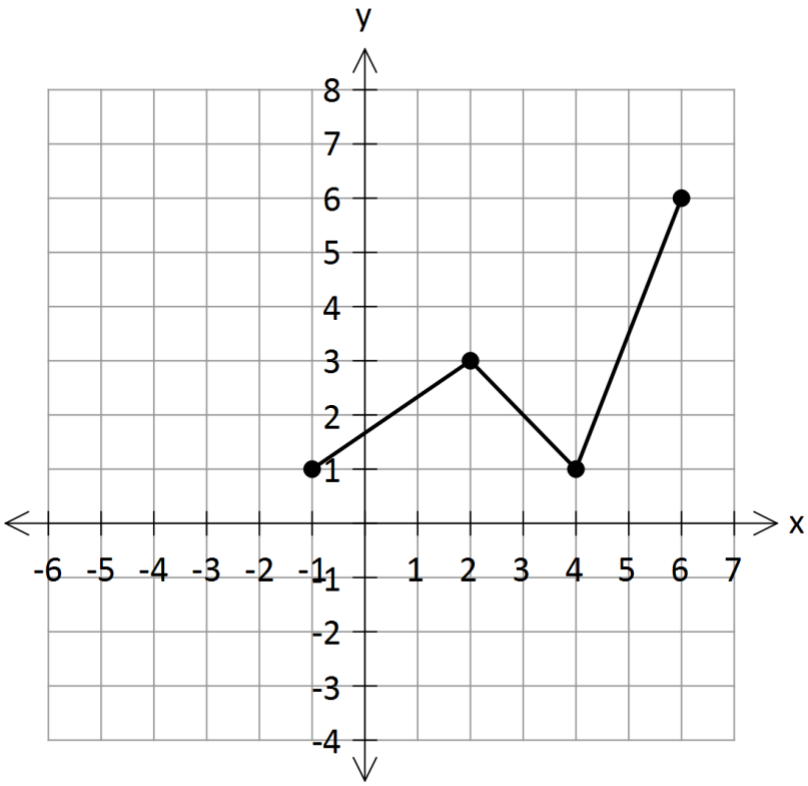
For a function $y = f(x)$, a vertical translation of the function is defined by the equation $y - k = f(x)$ where k is the number of units that the graph is shifted vertically.

For a function $y = f(x)$, a horizontal translation of the function is defined by the equation $y = f(x - h)$ where h is the number of units that the graph is shifted horizontally.

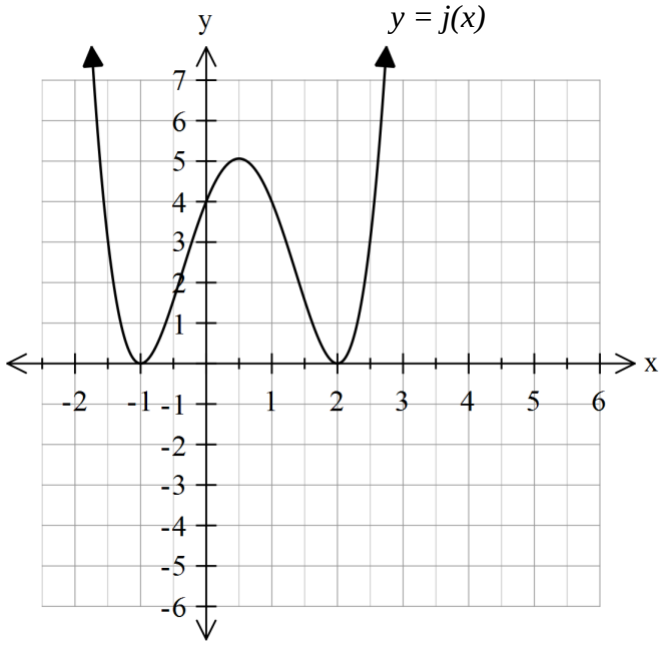
Example 1: Here is the graph of $y = g(x)$. For each of the translations given:

a) Sketch the image graph after each translation.
b) Write the equation of the image graph in terms of the function g.
c) State the domain and range of each function.

- A translation of 4 units left.
- A translation of 1 unit up.



Example 2: Here is the graph of $y = j(x)$. Sketch the image graph after a translation of 4 units right and 5 units down. Write the equation of the image graph in terms of the function j. State the domain and range of the original function and its image function.



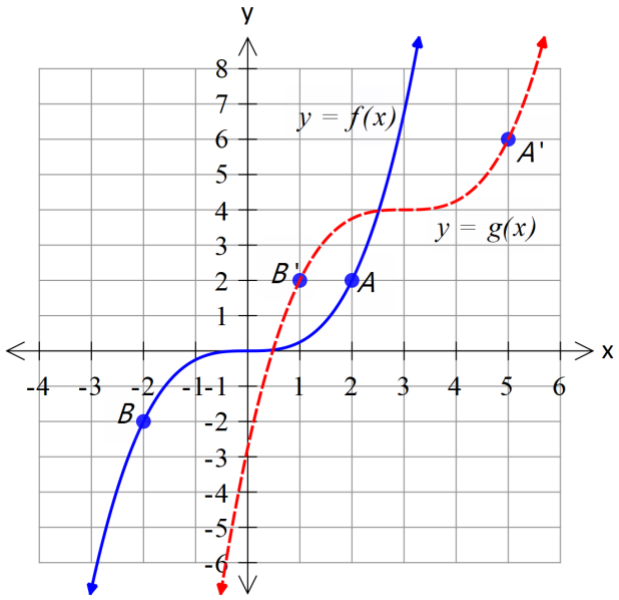
Example 3: Determining the coordinates of image points after translations.

a) The point A (2, 5) lies on the graph of $y = f(x)$.What are the coordinates of its image A′ on the graph of $y - 5 = f(x + 4)$

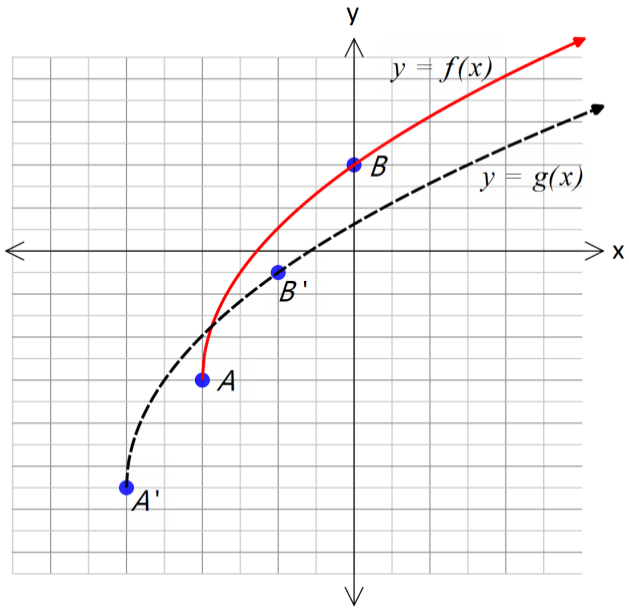
b) The point B (-1, 7) lies on the graph of $y = f(x)$.What are the coordinates of its image B′ on the graph of $y = f(x - 2) - 6$

Example 4:

a) The graph of $y = g(x)$ is the image of the graph of $y = f(x)$ after a transformation. Corresponding points are labelled. Write an equation of the image graph in terms of the function f.



b) The graph of $y = g(x)$ is the image of the graph of $y = f(x)$ after a transformation. Corresponding points are labelled. Write an equation of the image graph in terms of the function f.



Lesson 2: Horizontal and Vertical Stretches and Compressions

For a function $y = f(x)$, a vertical compression or stretch of the function is defined by the equation $y = af(x)$.

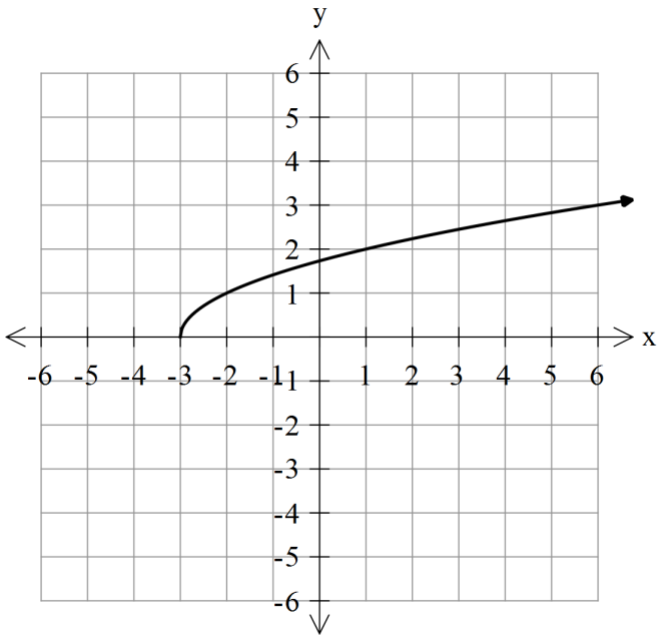
- If $0 < |a| < 1$, there is a vertical compression by a factor of $|a|$.
- If $|a| > 1$,there is a vertical stretch by a factor of $|a|$.
- If $a < 0$, there is reflection in the x −axis as well as the stretch or compression.

For a function$y = f(x)$, a horizontal stretch or compression of the function is defined by the equation $y = f(bx)$.

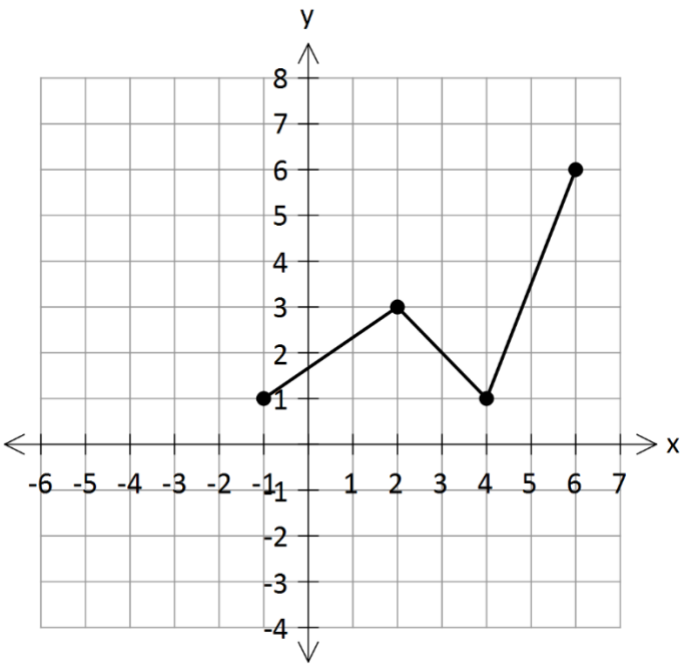
- If $0 < |b| < 1$, there is a horizontal stretch by a factor of $\frac{1}{|b|}$.
- If $|b| > 1$, there is a horizontal compression by a factor of $\frac{1}{|b|}$.
- If $b < 0$, there is a reflection in the y −axis as well as the stretch or compression.

Example 1: Given the graph of $y = f(x)$, sketch the graph of $y = 2f(x)$.

State the domain and range of the transformed function.

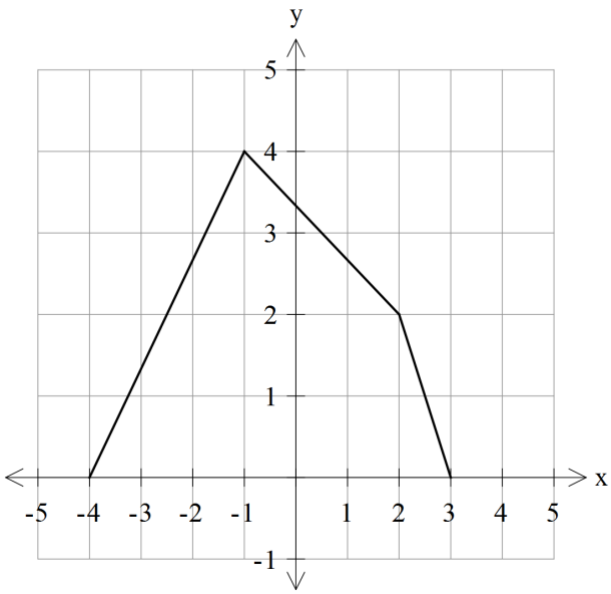


Example 2: Given the graph of $y = f(x)$, sketch the graph of $y = \frac{1}{3}f(x)$.

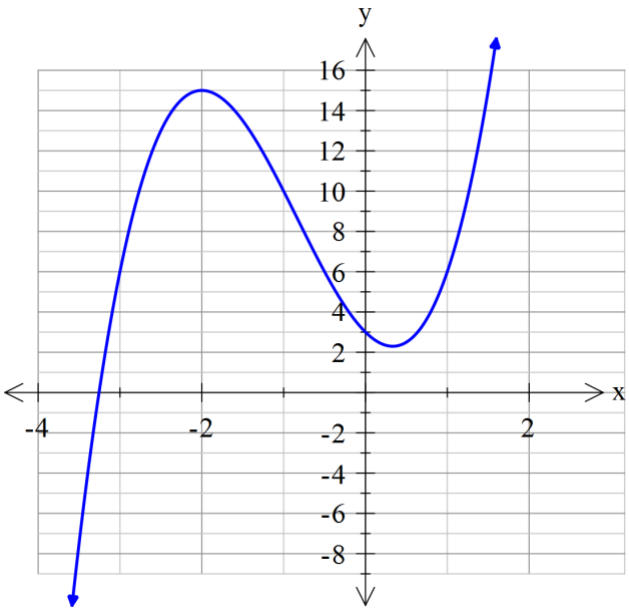


Example 3: Given the graph of $y = f(x)$. Sketch the graph of $y = f(2x)$

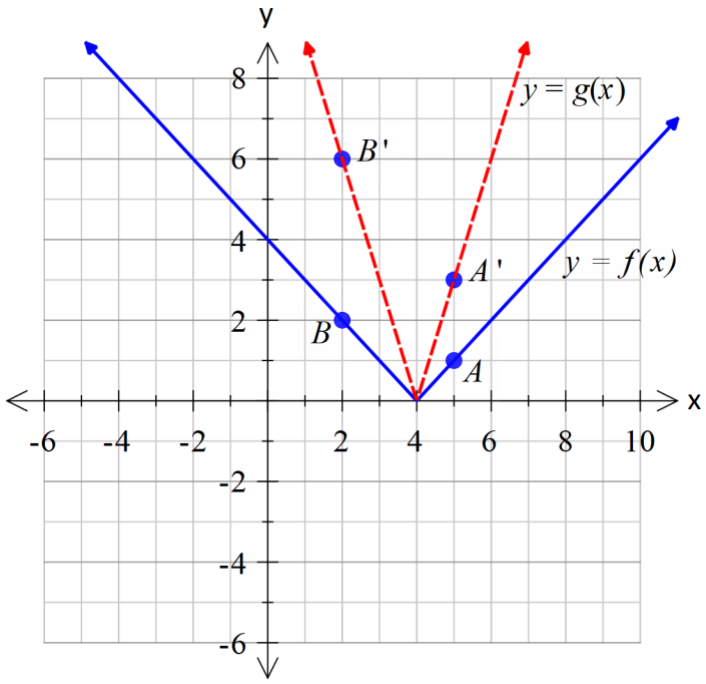
State the domain and range of the transformed function.



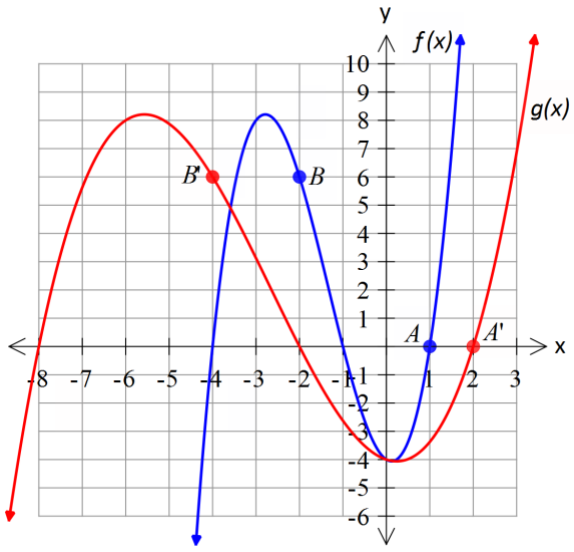
Example 4: Given the graph of $y = h(x)$. Sketch the graph of $y = -\frac{1}{2}h(2x)$.
State the domain and range of the transformed function.



Example 5: The graph of $y = g(x)$ is the image of the graph of $y = f(x)$ after a transformation. Corresponding points are labelled. Write an equation of the image graph in terms of the function f.

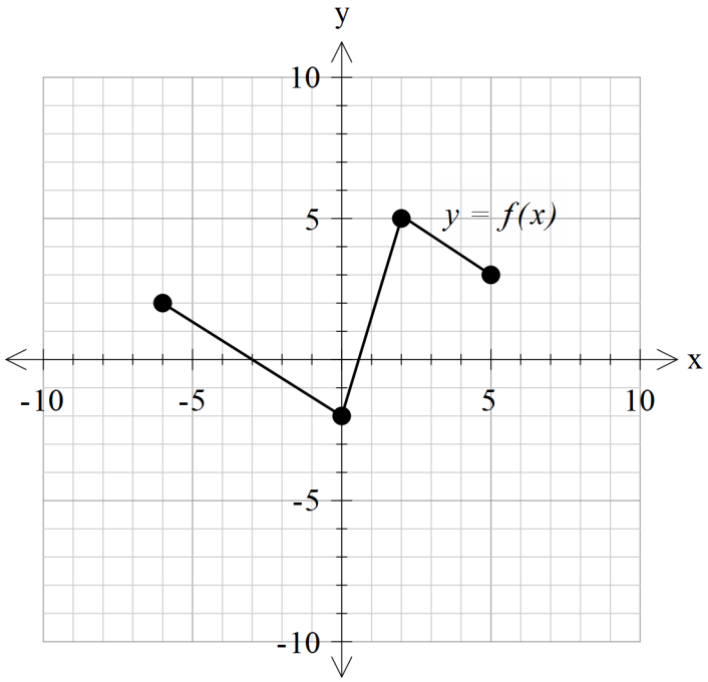


Example 6: The graph of $y = g(x)$ is the image of the graph of $y = f(x)$ after a transformation. Corresponding points are labelled. Write an equation of the image graph in terms of the function f.

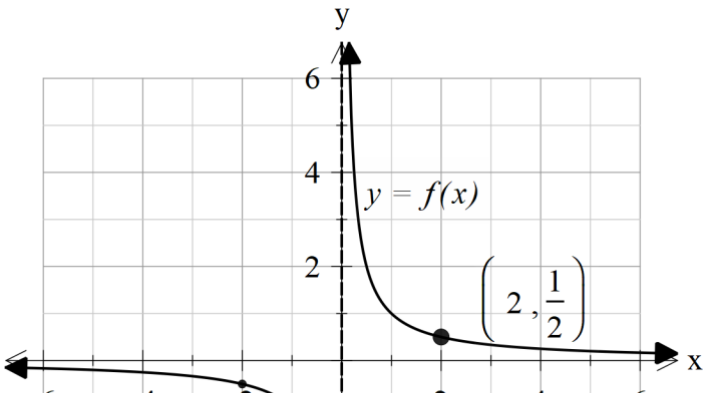


Lesson 3: Combinations of Transformations

Example 1: Given the graph of $y = f(x)$. Sketch its image after a horizontal compression by a factor of $\frac{1}{2}$, then a translation of 3 units down. Write the equation of the new function in terms of $f(x)$.

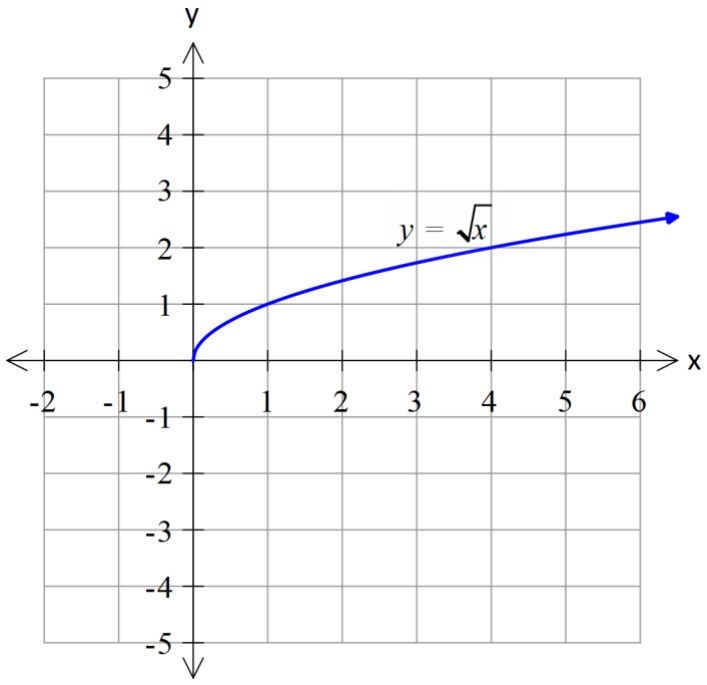


Example 3:
Given the graph of $y = f(x)$. Describe and sketch the graph of the transformations represented by $y + 4 = f(\frac{1}{2}(x + 1))$.



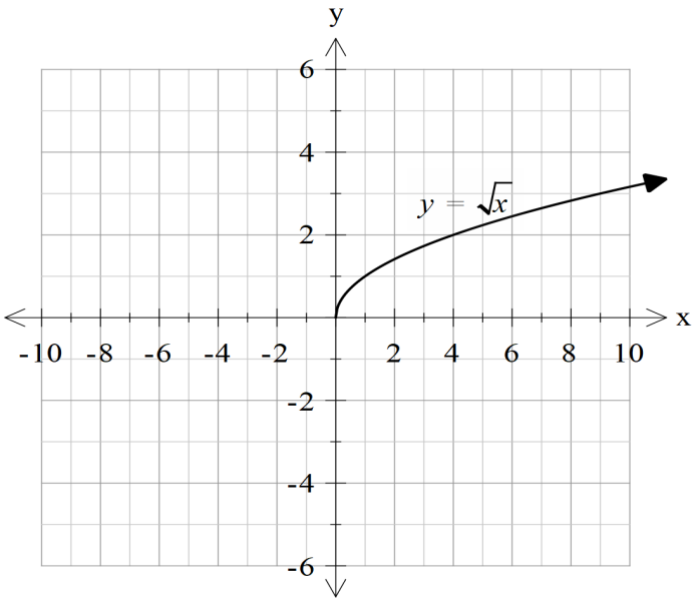
Example 3:

Given the graph of $y = \sqrt{x}$. Describe and sketch the graph of the transformations represented by $y - 2 = -\sqrt{3x + 3}$.



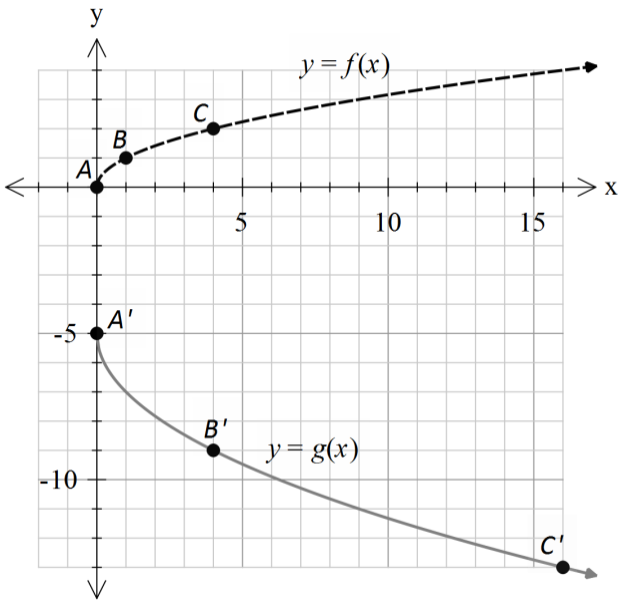
Example 4:

Given the graph of $y = \sqrt{x}$. Describe and sketch the graph of $y + 3 = 2\sqrt{-x - 4}$.



Example 5:

a) Determine as equation for $g(x)$ of the form $y - k = af(b(x - h))$ given the graphs of $y = f(x)$ and of the transformed function $y = g(x)$.



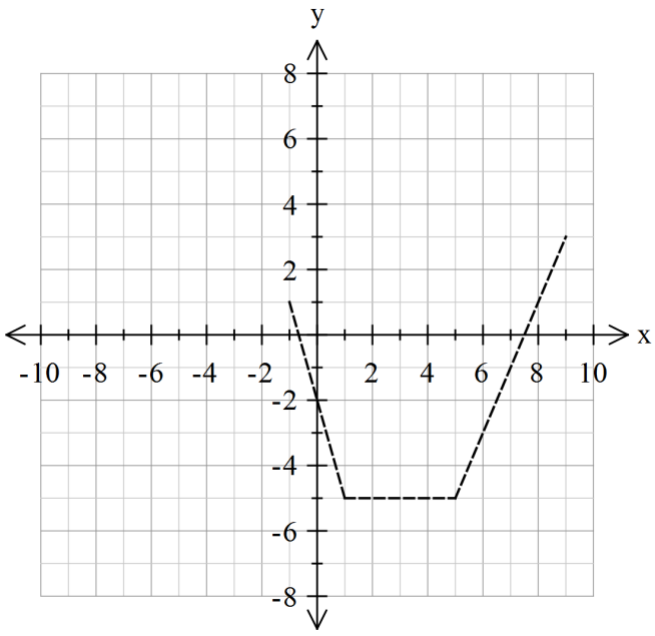
Assignment Time! Work on p.226- 3 – 10 (not 10c), 12, MC 1&2

Lesson 4: Reflections

A function can be reflected in the x −axis, the y −axis, or the line $y = x$. Note that a reflections in the y −axis results in a change in the signs of the x coordinate of a point and a reflection in the x −axis results in a change in the sign of the y coordinate of a point.

Example 1

a) Given the graph of a function shown below. Sketch the image graph after a reflection in the y-axis. State the domain and range of each function.



Example 2

a) What is an equation of the image when the graph of $y = \sqrt{x} + 5$ is reflected in the y-axis?

b) What is an equation of the image when the graph of $y = |x - 3|$ is reflected in the x-axis?

c) Write an equation for the image of the function $g(x) = -4x^2 + 3x - 7$ if it is:

i) reflected in the x-axis

ii) reflected in the y-axis

Example 3

The function $y = f(x)$ has domain $3 \leq x \leq 12$ and range $-9 \leq y \leq 2$. Determine the domain and range of each function.

a) $y = -f(x)$

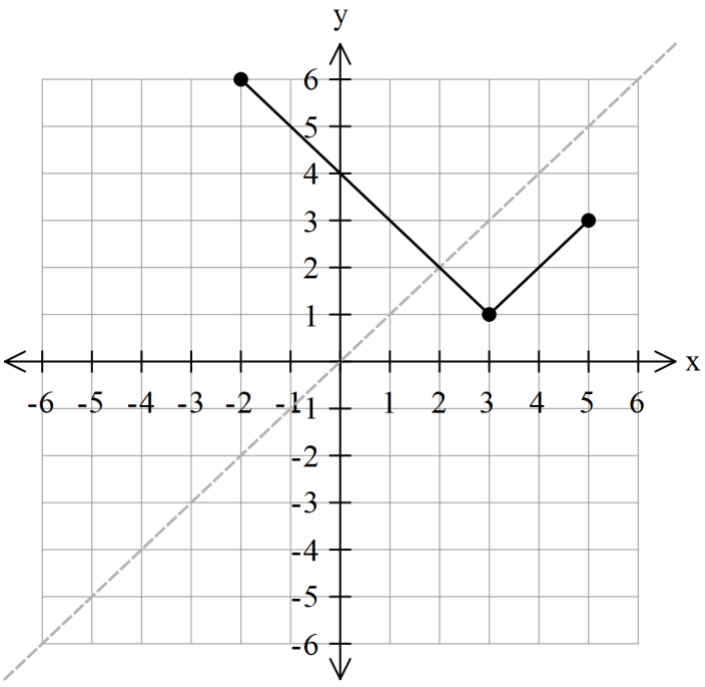
b) $y = f(-x)$

Assignment Time! Work on p.183- 3-5, 7-12, MC 1&2

Lesson 5: Inverses of Relations

Functions that are reflections of each other in the line $y = x$ are called inverses of each other. A reflection the in the line $y = x$ results in the x and y values "switching" places. The inverse of a function $f(x)$ is denoted by the notation $f^{-1}(x)$.

Example 1:
Graph the inverse of the function $f(x)$ shown below.



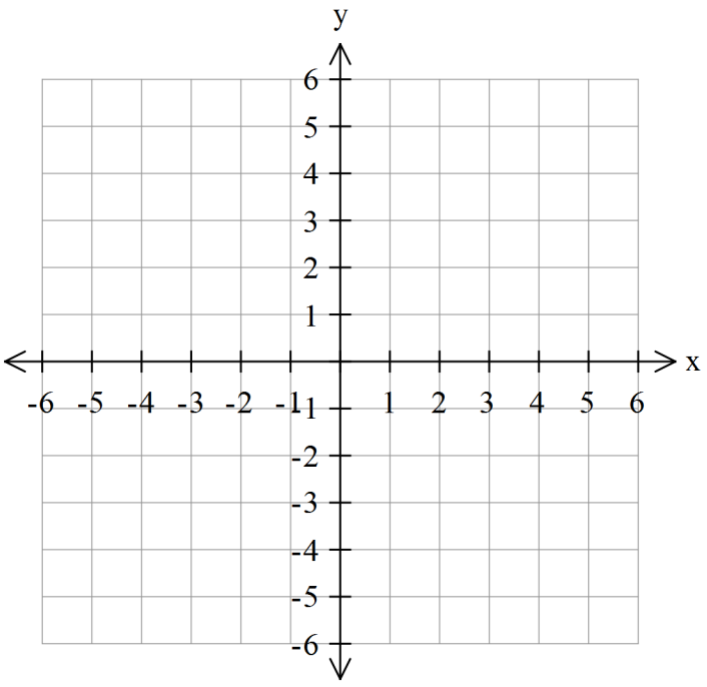
Domain and Range of $f(x)$:

Domain and Range of $f^{-1}(x)$.

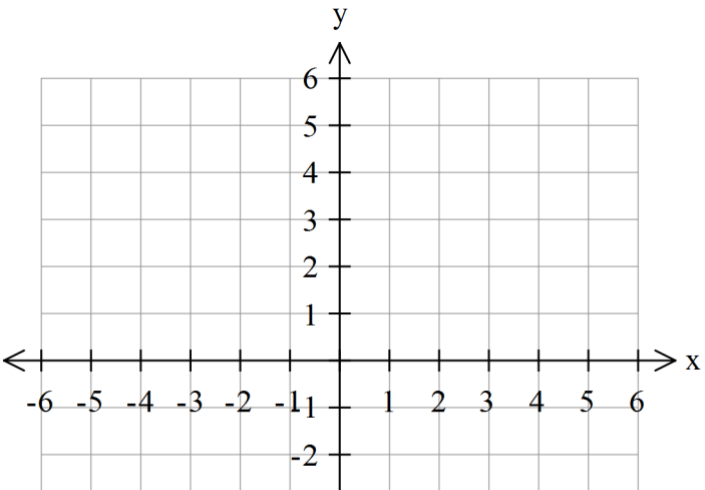
Example 2:

Algebraically determine the equation of the inverse of each function. Sketch graphs of $f(x)$ and its inverse.

a) $f(x) = \frac{1}{2}x + 1$



b) $y = (x-2)^2 + 5$



Example 3:

a) Determine an equation of the inverse of $y = (x - 1)^2 + 2$

b) Sketch graph of the function and its inverse.

c) Is the inverse a function? Explain.