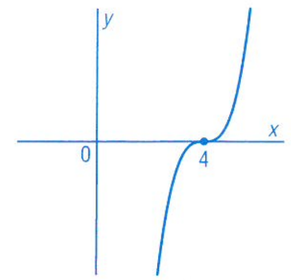
b) Is there more than one possible equation for each function in part a? Explain.

Yes, if I multiply the polynomial by a constant factor, I don't change the zeros but I do change the equation.

12. Sketch a possible graph of each polynomial function.

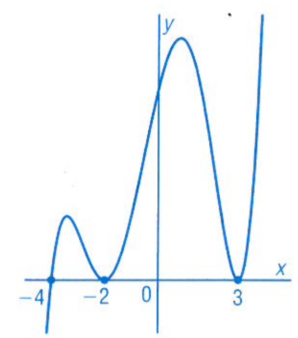
a) cubic function; leading coefficient is positive; zero of 4 has multiplicity 3

The zero has multiplicity 3, so the graph crosses the x-axis at $x = 4$. Since the function is cubic, there are no more zeros. The leading coefficient is positive so as $x \to -\infty$, the graph falls and as $x \to \infty$, the graph rises.



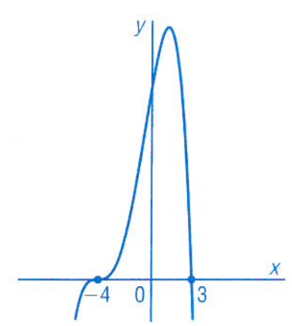
b) quintic function; leading coefficient is positive; zero of 3 has multiplicity 2; zero of −2 has multiplicity 2; zero of −4 has multiplicity 1

Each of the zeros 3 and −2 has multiplicity 2, so the graph just touches the x-axis at $x = 3$ and $x = -2$. The zero −4 has multiplicity 1, so the graph crosses the x-axis at $x = -4$. Since the function is quintic, there are no more zeros. The leading coefficient is positive, so as $x \to -\infty$, the graph falls and as $x \to \infty$, the graph rises.



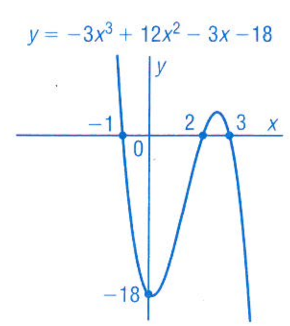
c) quartic function; leading coefficient is negative; zero of −4 has multiplicity 3; zero of 3 has multiplicity 1

The zero −4 has multiplicity 3, so the graph crosses the x-axis at $x = -4$. The zero 3 has multiplicity 1, so the graph crosses the x-axis at $x = 3$. Since the function is quartic, there are no more zeros. The leading coefficient is negative, so the graph opens down.



13. A cubic function has zeros 2, 3, and −1. The y-intercept of its graph is −18. Sketch the graph, then determine an equation of the function.

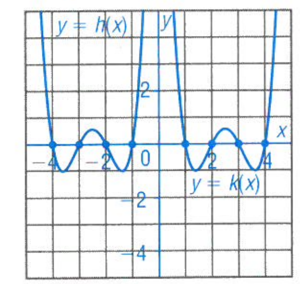
The zeros of the function are the roots of its equation.
Let k represent the leading coefficient.
$y = k(x - 2)(x - 3)(x + 1)$
The constant term in the equation is −18.
So, $k(-2)(-3)(1) = -18$
$k = -3$
So, an equation is:
$y = -3(x - 2)(x - 3)(x + 1)$
$y = -3(x^2 - 5x + 6)(x + 1)$
$y = -3x^3 + 12x^2 - 3x - 18$



C

14. Investigate pairs of graphs of even-degree polynomial functions of the form shown below for different values of the variables a, b, c, and $d \in \mathbb{Z}$. Describe one graph as a transformation image of the other graph. What conclusions can you make?
$h(x) = (x + a)(x + b)(x + c)(x + d)$ and
$k(x) = (x - a)(x - b)(x - c)(x - d)$

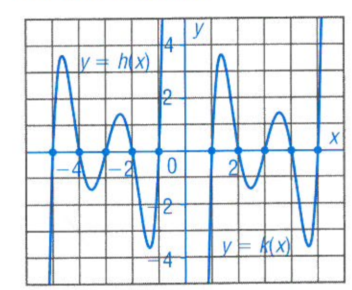
Sample response:



The graph of $k(x) = (x - 1)(x - 2)(x - 3)(x - 4)$ is the image of the graph of $h(x) = (x + 1)(x + 2)(x + 3)(x + 4)$ after a reflection in the y-axis. In general, the graph of $k(x)$ is the image of the graph of $h(x)$ after a reflection in the y-axis.

15. Investigate pairs of graphs of odd-degree polynomial functions of the form shown below for different values of the variables a, b, c, d, and $e \in \mathbb{Z}$. Describe one graph as a transformation image of the other graph. What conclusions can you make?
$h(x) = (x + a)(x + b)(x + c)(x + d)(x + e)$ and
$k(x) = (x - a)(x - b)(x - c)(x - d)(x - e)$

Sample response:



The graph of
$k(x) = (x - 1)(x - 2)(x - 3)(x - 4)(x - 5)$ is the image of the graph of $h(x) = (x + 1)(x + 2)(x + 3)(x + 4)(x + 5)$ after a rotation of 180° about the origin. In general, the graph of $k(x)$ is the image of the graph of $h(x)$ after a rotation of 180° about the origin.