8. Identify the graph that corresponds to each function. Justify your choices.

a) $f(x) = -x^3 + 3x^2 + x - 3$

Odd degree, negative leading coefficient: graph B

b) $g(x) = x^4 - 3x^2 - 3$

Even degree, positive leading coefficient: graph D

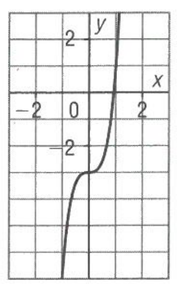
c) $h(x) = x^5 + 3x^3 - 3$

Odd degree, positive leading coefficient: graph A

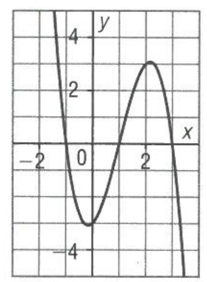
d) $k(x) = -x^2 + 4x - 3$

Even degree, negative leading coefficient: graph C

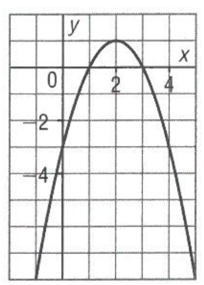
i) Graph A



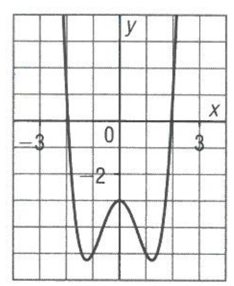
ii) Graph B



iii) Graph C



iv) Graph D



9. Determine the zeros of each polynomial function. State the multiplicity of each zero. How does the graph of each function behave at the related x-intercepts?
Use graphing technology to check.

a) $f(x) = (x + 3)^3$

$0 = (x + 3)^3$
Root of the equation: $x = -3$
Zero of the function: -3
The zero has multiplicity 3.
So, the graph crosses the x-axis at $x = -3$.

b) $g(x) = (x - 2)^2(x + 3)^2$

$0 = (x - 2)^2(x + 3)^2$
Roots of the equation: $x = 2$ and $x = -3$
Zeros of the function: 2 and -3
The zero 2 has multiplicity 2.
The zero -3 has multiplicity 2.
So, the graph just touches the x-axis at $x = 2$ and at $x = -3$.

TEACHER NOTE

Achievement Indicator

Question 9 addresses
AI 12.5: Explain how the multiplicity of a zero of a polynomial function affects the graph.

c) $h(x) = (x - 1)^4(2x + 1)$

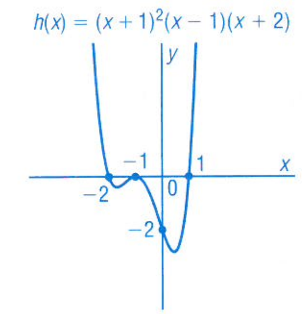
$0 = (x - 1)^4(2x + 1)$
Roots of the equation: $x = 1$ and $x = -0.5$
Zeros of the function: 1 and -0.5
The zero 1 has multiplicity 4.
The zero -0.5 has multiplicity 1.
So, the graph just touches the x-axis at $x = 1$ and crosses the x-axis at $x = -0.5$.

d) $j(x) = (x - 4)^3(x + 1)^2$

$0 = (x - 4)^3(x + 1)^2$
Roots of the equation: $x = 4$ and $x = -1$
Zeros of the function: 4 and -1
The zero 4 has multiplicity 3.
The zero -1 has multiplicity 2.
So, the graph crosses the x-axis at $x = 4$ and just touches the x-axis at $x = -1$.

10. Sketch the graph of this polynomial function.
$h(x) = (x + 1)^2(x - 1)(x + 2)$

To determine the roots, let $h(x) = 0$.
$0 = (x + 1)^2(x - 1)(x + 2)$
Zeros of the function: -1, 1, and -2
The zero -1 has multiplicity 2.
The zeros 1 and -2 have multiplicity 1.
So, the graph just touches the x-axis at $x = -1$ and crosses the x-axis at $x = 1$ and at $x = -2$.
The equation has degree 4, so it is an even-degree polynomial function.
The leading coefficient is positive, so the graph opens up. The y-intercept is:
$(1)^2(-1)(2) = -2$



TEACHER NOTE

Achievement Indicators

Question 10 addresses
AI 12.2: Explain the role of the constant term and leading coefficient in the equation of a polynomial function with respect to the graph of the function.
AI 12.5: Explain how the multiplicity of a zero of a polynomial function affects the graph.
AI 12.6: Sketch, with or without technology, the graph of a polynomial function.

11. a) Write an equation in standard form for each polynomial function described below.

i) a cubic function with zeros 3, -3, and 0

Sample response:
The zeros of the function are the roots of its equation.
$y = x(x - 3)(x + 3)$
$y = x(x^2 - 9)$
$y = x^3 - 9x$

ii) a quartic function with zeros -2 and 1 of multiplicity 1, and a zero 2 of multiplicity 2

Sample response:
The zeros of the function are the roots of its equation.
$y = (x + 2)(x - 1)(x - 2)^2$
$y = (x^2 + x - 2)(x^2 - 4x + 4)$
$y = x^4 - 4x^3 + 4x^2 + x^3 - 4x^2 + 4x - 2x^2 + 8x - 8$
$y = x^4 - 3x^3 - 2x^2 + 12x - 8$