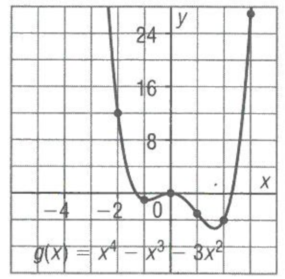


x	$g(x)$
−2	12
−1	−1
0	0
1	−3
2	−4
3	27

Example 2 Using Intercepts to Sketch the Graph of a Polynomial Function

Sketch the graph of the polynomial function:
$f(x) = 2x^4 - x^3 - 14x^2 + 19x - 6$

SOLUTION

Factor the polynomial. Use the factor theorem.
List the factors of the constant term, −6:
$1, -1, 2, -2, 3, -3, 6, -6$
Use mental math. When $x = 1, f(1) = 0$
So, $x - 1$ is a factor of $2x^4 - x^3 - 14x^2 + 19x - 6$.
Divide to determine the other factor.

1	2	−1	−14	19	−6
		2	1	−13	6
	2	1	−13	6	0

So, $2x^4 - x^3 - 14x^2 + 19x - 6 = (x - 1)(2x^3 + x^2 - 13x + 6)$
Factor the cubic polynomial. Use the factor theorem.
Let $P(x) = 2x^3 + x^2 - 13x + 6$
Use mental math.
When $x = 1, P(1) \neq 0$
When $x = -1, P(-1) \neq 0$
So, neither $x - 1$ nor $x + 1$ is a factor.
Try $x = 2$: $P(2) = 2(2)^3 + (2)^2 - 13(2) + 6$
$= 16 + 4 - 26 + 6$
$= 0$
So, $x - 2$ is a factor of $2x^3 + x^2 - 13x + 6$.
Divide to determine the other factor.

2	2	1	−13	6
		4	10	−6
	2	5	−3	0

$2x^4 - x^3 - 14x^2 + 19x - 6 = (x - 1)(x - 2)(2x^2 + 5x - 3)$

TEACHER NOTE

DI: Extending Thinking
Introduce the concepts of *absolute maximum point* and *absolute minimum point*. These points only occur on the graphs of even-degree polynomial functions.

Check Your Understanding

2. Sketch the graph of the polynomial function:
$f(x) = x^3 + x^2 - 6x$

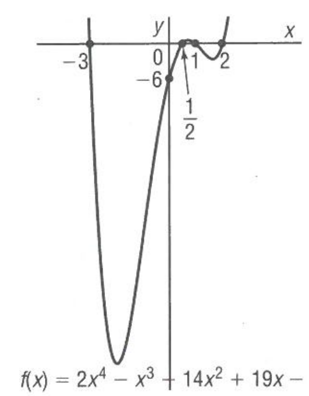
Factor. $f(x) = x(x^2 + x - 6)$
$f(x) = x(x + 3)(x - 2)$
Determine the zeros of $f(x)$.
Let $f(x) = 0$.
$0 = x(x + 3)(x - 2)$
So, $x = 0$ or $x + 3 = 0$ or $x - 2 = 0$
$x = -3$ $x = 2$
The zeros are: 0, −3, 2
So, the x-intercepts of the graph are: 0, −3, and 2
Plot points at the intercepts.
The equation has degree 3, so it is an odd-degree polynomial function.
The leading coefficient is positive, so as $x \to -\infty$, the graph falls and as $x \to \infty$, the graph rises.
The constant term is 0, so the y-intercept is 0.
Draw a smooth curve through the points, beginning at the bottom left and ending at the top right.

TEACHER NOTE

Tell students that when sketching a graph, they need label only the intercepts.

DO NOT COPY. ©P

Factor the trinomial: $2x^2 + 5x - 3 = (x + 3)(2x - 1)$
So, $f(x) = (x - 1)(x - 2)(x + 3)(2x - 1)$
Determine the zeros of $f(x)$. Let $f(x) = 0$.
$0 = (x - 1)(x - 2)(x + 3)(2x - 1)$ Solve the equation.
So, $x - 1 = 0$ or $x - 2 = 0$ or $x + 3 = 0$ or $2x - 1 = 0$
$x = 1$ $x = 2$ $x = -3$ $x = \frac{1}{2}$
The zeros are: $1, 2, -3, \frac{1}{2}$
So, the x-intercepts of the graph are: $1, 2, -3$, and $\frac{1}{2}$
Plot points at the intercepts.
The equation has degree 4, so it is an even-degree polynomial function. The leading coefficient is positive, so the graph opens up.
The constant term is −6, so the y-intercept is −6.
Draw a smooth curve through the points, beginning at the top left and ending at the top right.



TEACHER NOTE

DI: Extending Thinking
Ask students how they would proceed with the solution in *Example 2* if the trinomial did not factor. (*Use the quadratic formula to determine the x-intercepts, if they exist.*)

THINK FURTHER

Suppose the graph of a polynomial function is symmetrical about the y-axis. What do you know about the function?

If the graph is symmetrical about the y-axis, the function is an even-degree function.

In *Example 2*, the equation $0 = (x - 1)(x - 2)(x + 3)(2x - 1)$ is the factored form of a **polynomial equation**. This *Example* illustrates that when the equation of a polynomial function is factorable, the x-intercepts of its graph can be determined by factoring.
The x-intercepts are the zeros of the polynomial function because they are the values of x when the function is 0. The zeros of the function are the roots of the related polynomial equation.

©P DO NOT COPY.