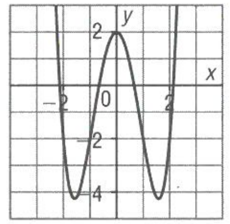
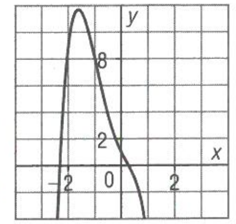
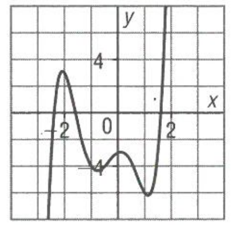
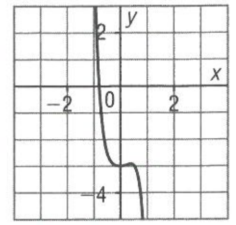
4	Quartic Even-degree polynomial function	$f(x) = x^4 - 5x^2 + 2$ 	$f(x) = -2x^4 - 3x^3 + 2x^2 - 4x + 1$ 
5	Quintic Odd-degree polynomial function	$f(x) = x^5 + 2x^4 - 3x^3 - 4x^2 + x - 3$ 	$f(x) = -7x^5 + x^2 - 3$ 

The end behaviour of a graph refers to the behaviour of the graph as $|x|$ becomes very large. For positive values of x, as x becomes very large, we say x approaches infinity. We write: $x \to \infty$
For negative values of x, as $|x|$ becomes very large, we say x approaches negative infinity. We write: $x \to -\infty$
The end behaviour of the graphs of polynomial functions can be summarized.

Odd-Degree Polynomial Functions

Leading Coefficient	End Behaviour of Graph
positive	As $x \to -\infty$, the graph falls to the left, and as $x \to \infty$, the graph rises to the right
negative	As $x \to -\infty$, the graph rises to the left, and as $x \to \infty$, the graph falls to the right

When a graph rises to the left and rises to the right, the graph *opens up*. When a graph falls to the left and falls to the right, the graph *opens down*.

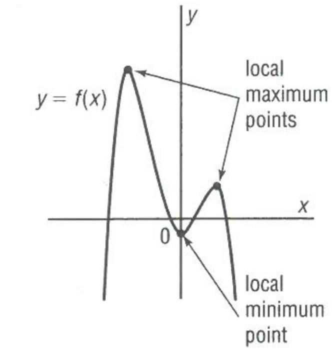
Even-Degree Polynomial Functions

Leading Coefficient	End Behaviour of Graph
positive	As $x \to -\infty$, the graph rises to the left, and as $x \to \infty$, the graph rises to the right
negative	As $x \to -\infty$, the graph falls to the left, and as $x \to \infty$, the graph falls to the right

For the graph of a polynomial function:

- A point where the graph changes from increasing to decreasing is called a **local maximum point**. The y-value of this point is greater than those of neighbouring points.

- A point where the graph changes from decreasing to increasing is called a **local minimum point**. The y-value of this point is less than those of neighbouring points.



An inspection of the graphs of polynomial functions in this lesson and in Lesson 1.3 illustrates that the graph of a polynomial function of degree n can have at most n x-intercepts and at most $(n - 1)$ local maximum or minimum points. For example, the graph of a cubic function can have at most 3 x-intercepts and at most 2 local maximum or local minimum points.

To sketch the graph of a polynomial function, use a table of values and knowledge of the end behaviour of its graph.

Extra Material

All graphs in solutions of *Check Your Understanding*

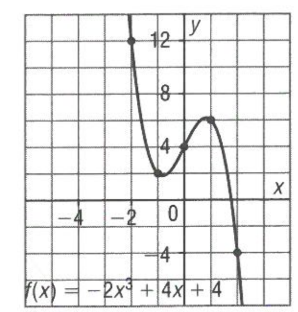
Example 1 Using a Table of Values to Sketch the Graph of a Polynomial Function

Sketch the graph of each polynomial function.

a) $f(x) = -2x^3 + 4x + 4$ **b)** $g(x) = x^4 - x^3 - 3x^2$

SOLUTION

a) The equation represents an odd-degree polynomial function. Since the leading coefficient is negative, as $x \to -\infty$, the graph rises and as $x \to \infty$, the graph falls. The constant term is 4, so the y-intercept is 4. Use a table of values to create the graph.



x	$f(x)$
−2	12
−1	2
0	4
1	6
2	−4

b) The equation represents an even-degree polynomial function. Since the leading coefficient is positive, the graph opens up. The constant term is 0, so the y-intercept is 0. Use a table of values to create the graph.

Check Your Understanding

1. Sketch the graph of each polynomial function.

a) $f(x) = x^3 - 3x^2$

b) $g(x) = -x^4 - 6x^3 - 9x^2 + 3$

a) The equation represents an odd-degree polynomial function. The leading coefficient is positive, so as $x \to -\infty$, the graph falls and as $x \to \infty$, the graph rises. The constant term is 0, so the y-intercept is 0.

x	$f(x)$
−2	−20
−1	−4
0	0
1	−2
2	−4
3	0
4	16

b) The equation represents an even-degree polynomial function. The leading coefficient is negative, so the graph opens down. The constant term is 3, so the y-intercept is 3.

x	$g(x)$
−4	−13
−3	3
−2	−1
−1	−1
0	3

Remember that a sketch does not have to be accurate.