

TEACHER NOTE

Have students verify their answers to *Check Your Understanding* by multiplying.

TEACHER NOTE

Remind students that the highest power in a polynomial is its degree.

To verify the answer in *Example 1*, multiply the quotient by the divisor, then add the remainder.


$$\begin{aligned}(x + 2)(3x^2 - 4x + 7) + (-20) \\ &= x(3x^2 - 4x + 7) + 2(3x^2 - 4x + 7) - 20 \\ &= 3x^3 - 4x^2 + 7x + 6x^2 - 8x + 14 - 20 \\ &= 3x^3 + 2x^2 - x - 6\end{aligned}$$

Since this is the original polynomial, the answer is correct.

In *Example 1*, note that the dividend is a polynomial of degree 3, and the quotient is a polynomial of degree 2. In general, when a polynomial is divided by a divisor of the form $x - a$, the degree of the quotient is 1 less than the degree of the dividend.

THINK FURTHER

When is it not necessary to use long division to divide a polynomial by a binomial?

 It is not necessary to use long division when the binomial is a factor of the polynomial. For example, to divide $3x^2 - 13x - 10$ by $x - 5$, I can factor the polynomial, then divide by the common factor: $3x^2 - 13x - 10 = (3x + 2)(x - 5)$ and $\frac{(3x + 2)\cancel{(x - 5)}}{\cancel{(x - 5)}} = 3x + 2$

When a polynomial is divided by a binomial, the *division statement* relates the original polynomial and the divisor to the quotient and remainder. This is how the answer to the polynomial division in *Example 1* was verified.

TEACHER NOTE

Ensure students understand that they can stop dividing when the degree of the remainder is less than that of the divisor.

Division Statement for Division by $x - a$

$P(x) = (x - a)Q(x) + R$, where $P(x)$ is the original polynomial, $(x - a)$ is the binomial divisor, $Q(x)$ is the quotient polynomial (which has degree 1 less than $P(x)$), and R is the remainder (which is a constant).

THINK FURTHER

In the division statement, when $R = 0$, how are $P(x)$ and $(x - a)$ related?

 $(x - a)$ is a factor of $P(x)$.

TEACHER NOTE

You may wish to have students investigate dividing into a polynomial with "missing" terms before it is presented in *Example 2*.

The polynomial may have some powers of the variable missing. Since adding 0 to a polynomial does not change its value, terms with coefficient 0 are inserted to hold the places of missing terms.