TEACHER NOTE

DI: Common Difficulties

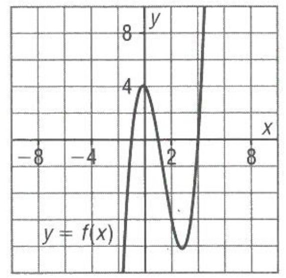
Students may want to use technology to match the equations with their graphs. Instruct students to use their conclusions from the preceding lesson; they can use technology to check their answers.

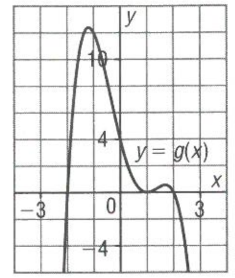
Construct Understanding

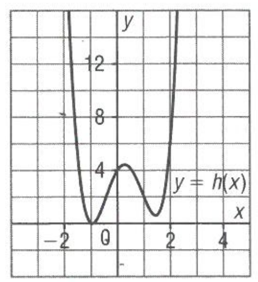
Match each graph with its equation.
Justify your choices.

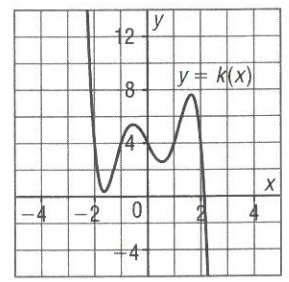
$y = -x^5 + 5x^3 - 4x + 4$ $\quad$ $y = 2x^4 - 2x^3 - 5x^2 + 3x + 4$

$y = x^3 - 4x^2 - x + 4$ $\quad$ $y = -x^4 + 2x^3 + 3x^2 - 8x + 4$









The constant term in each equation is 4, so all the graphs have y-intercept 4. The equation $y = -x^5 + 5x^3 - 4x + 4$ represents a quintic function. Since the graph of $y = k(x)$ has 2 hills and 2 valleys, the graph is that of a quintic function. So, $y = -x^5 + 5x^3 - 4x + 4$ matches the graph of $y = k(x)$.

The equation $y = 2x^4 - 2x^3 - 5x^2 + 3x + 4$ represents a quartic function. Since the coefficient of x^4 is positive, its graph opens up. This matches the graph of $y = h(x)$.

The equation $y = x^3 - 4x^2 - x + 4$ represents a cubic function. Since the coefficient of x^3 is positive, its graph falls to the left and rises to the right. This matches the graph of $y = f(x)$.

The equation $y = -x^4 + 2x^3 + 3x^2 - 8x + 4$ represents a quartic function. Since the coefficient of x^4 is negative, its graph opens down. This matches the graph of $y = g(x)$.

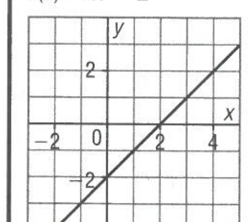
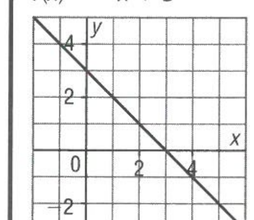
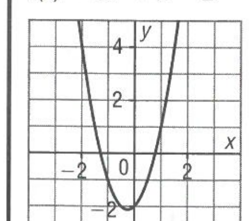
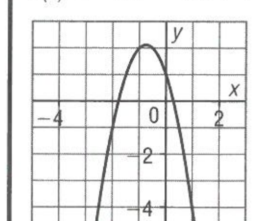
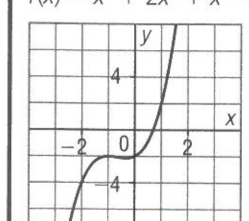
Linear, quadratic, cubic, quartic, and quintic functions are polynomial functions.
The *degree* of a polynomial function is the highest power of the variable in the equation. For example, the function $f(x) = -2x^3 + 3x^5 - 3 + x^2$ has degree 5 because the highest power of x is x^5.

Polynomial Functions

A polynomial function of degree n can be written in standard form as:
$f(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1x + a_0$, where n is a whole number and $a_n, a_{n-1}, a_{n-2}, \ldots, a_1, a_0$ are real numbers.

The coefficient of the highest power of x is the **leading coefficient.**

The graph of a polynomial function is smooth and continuous, which means it has no sharp corners and can be drawn without lifting the pencil from the paper.
A polynomial function can be described by its degree:
even-degree polynomial function or **odd-degree polynomial function.**

Degree	Type	Positive leading coefficient	Negative leading coefficient
1	Linear Odd-degree polynomial function	$f(x) = x - 2$ 	$f(x) = -x + 3$ 
2	Quadratic Even-degree polynomial function	$f(x) = 2x^2 + x - 2$ 	$f(x) = -2x^2 - 3x + 1$ 
3	Cubic Odd-degree polynomial function	$f(x) = x^3 + 2x^2 + x - 2$ 	$f(x) = -x^3 - 2x^2 + 5$