b) $x^4 - 5x^2 + 4$

Let $P(x) = x^4 - 5x^2 + 4$
The factors of 4 are: 1, −1, 2, −2, 4, −4
Use mental math to substitute $x = 1$:
$P(1) = 0$; so, $x - 1$ is a factor.
Use mental math to substitute $x = -1$:
$P(-1) = 0$; so, $x + 1$ is a factor.
Try $x = 2$: $P(2) = (2)^4 - 5(2)^2 + 4$
$= 0$
So, $x - 2$ is a factor.
Try $x = -2$: $P(-2) = (-2)^4 - 5(-2)^2 + 4$
$= 0$
So, $x + 2$ is a factor.
Since the original polynomial has degree 4, it can have at most 4 binomial factors.
So, $x^4 - 5x^2 + 4 = (x - 1)(x + 1)(x - 2)(x + 2)$

12. a) What value of b will ensure $x + 3$ is a factor of $bx^3 - 2x^2 + x - 6$?

Let $P(x) = bx^3 - 2x^2 + x - 6$
If $x + 3$ is a factor, $P(-3) = 0$
$P(-3) = b(-3)^3 - 2(-3)^2 + (-3) - 6$
$= -27b - 27$
Let $P(-3) = 0$
$-27b - 27 = 0$
$b = -1$
So, the value of b is −1.

b) What value of d will ensure $x + 2$ is a factor of $3x^5 - dx^4 + 4x^3 - 2dx^2 + x + 10$?

Let $P(x) = 3x^5 - dx^4 + 4x^3 - 2dx^2 + x + 10$
If $x + 2$ is a factor, $P(-2) = 0$
$P(-2) = 3(-2)^5 - d(-2)^4 + 4(-2)^3 - 2d(-2)^2 + (-2) + 10$
$= -120 - 24d$
Let $P(-2) = 0$
$-120 - 24d = 0$
$d = \frac{120}{-24}$, or -5
So, the value of d is −5.

13. Determine whether $x + b$ is a factor of $(x + b)^5 + (x + p)^5 + (b - p)^5$, $b, p \in \mathbb{R}$.

Let $P(x) = (x + b)^5 + (x + p)^5 + (b - p)^5$
If $x + b$ is a factor, $P(-b) = 0$
$P(-b) = (-b + b)^5 + (-b + p)^5 + (b - p)^5$
$= 0 + (-(b - p))^5 + (b - p)^5$
$= 0 - (b - p)^5 + (b - p)^5$
$= 0$
Since the remainder is 0, $(x + b)$ is a factor of $(x + b)^5 + (x + p)^5 + (b - p)^5$, $b, p \in \mathbb{R}$.

C

14. When $mx^3 - 2x^2 + nx - 4$ is divided by $x + 2$, the remainder is 4. When $mx^3 - 2x^2 + nx - 4$ is divided by $x - 1$, the remainder is −11. Determine the values of m and n.

Let $P(x) = mx^3 - 2x^2 + nx - 4$
$P(-2) = 4$
$P(-2) = m(-2)^3 - 2(-2)^2 + n(-2) - 4$
$4 = -8m - 2n - 12$
$0 = -8m - 2n - 16$ ①
$P(1) = -11$
$P(1) = m(1)^3 - 2(1)^2 + n(1) - 4$
$-11 = m + n - 6$
$0 = m + n + 5$ ②
Solve the system of equations:
$0 = -8m - 2n - 16$ ①
$0 = m + n + 5$ ②
Solve equation ② for m: $m = -n - 5$
Substitute for m in equation ①.
$0 = -8m - 2n - 16$ ①
$0 = -8(-n - 5) - 2n - 16$
$0 = 8n + 40 - 2n - 16$
$0 = 6n + 24$
$n = -4$
Substitute $n = -4$ in equation ②.
$0 = m - 4 + 5$
$m = -1$
So, $m = -1$ and $n = -4$

15. Determine each remainder.

a) $(8x^2 - 6x + 3) \div (4x + 1)$

$$\begin{array}{r} 2x - 2 \\ 4x + 1 \overline{) 8x^2 - 6x + 3} \\ \underline{8x^2 + 2x} \\ -8x + 3 \\ \underline{-8x - 2} \\ 5 \end{array}$$

The remainder is 5.

b) $(3x^3 + 2x^2 - 6x - 1) \div (3x + 2)$

$$\begin{array}{r} x^2 \quad - 2 \\ 3x + 2 \overline{) 3x^3 + 2x^2 - 6x - 1} \\ \underline{3x^3 + 2x^2} \\ 0 - 6x - 1 \\ \underline{- 6x - 4} \\ 3 \end{array}$$

The remainder is 3.