7. Determine the remainder.

a) $(2x^3 - x^2 + 3x - 2) \div (x - 3)$

Let $P(x) = 2x^3 - x^2 + 3x - 2$
$P(3) = 2(3)^3 - (3)^2 + 3(3) - 2$
$= 54 - 9 + 9 - 2$
$= 52$
The remainder is 52.

b) $(3x^3 - 2x^2 - 4x + 6) \div (x - 2)$

Let $P(x) = 3x^3 - 2x^2 - 4x + 6$
$P(2) = 3(2)^3 - 2(2)^2 - 4(2) + 6$
$= 24 - 8 - 8 + 6$
$= 14$
The remainder is 14.

8. When $2x^3 + kx^2 - 3x + 2$ is divided by $x - 2$, the remainder is 4. Determine the value of k.

Let $P(x) = 2x^3 + kx^2 - 3x + 2$
$P(2) = 2(2)^3 + k(2)^2 - 3(2) + 2$
$= 16 + 4k - 6 + 2$
$= 12 + 4k$
The remainder is 4.
So, $12 + 4k = 4$ Solve for k.
$4k = -8$
$k = -2$
The value of k is -2.

9. Determine one binomial factor of each polynomial.

a) $x^4 + 6x^3 + 5x^2 - 24x - 36$

Sample response:
Let $P(x) = x^4 + 6x^3 + 5x^2 - 24x - 36$
The factors of -36 are: 1, -1, 2, -2, 3, -3, 4, -4, 6, -6, 9, -9, 12, -12, 18, -18, 36, -36
Use mental math to substitute $x = 1$, then $x = -1$ to determine that neither $x - 1$ nor $x + 1$ is a factor.
Try $x = 2$: $P(2) = (2)^4 + 6(2)^3 + 5(2)^2 - 24(2) - 36$
$= 0$
So, $x - 2$ is a factor of $x^4 + 6x^3 + 5x^2 - 24x - 36$.

b) $x^5 + 3x^4 - 5x^3 - 15x^2 + 4x + 12$

Sample response:
Let $P(x) = x^5 + 3x^4 - 5x^3 - 15x^2 + 4x + 12$
The factors of 12 are: 1, -1, 2, -2, 3, -3, 4, -4, 6, -6, 12, -12
Use mental math to substitute $x = 1$:
$P(1) = 0$
So, $x - 1$ is a factor of $x^5 + 3x^4 - 5x^3 - 15x^2 + 4x + 12$.

10. a) Show that $x + 5$ is a factor of $x^3 + 4x^2 - 11x - 30$.

Let $P(x) = x^3 + 4x^2 - 11x - 30$
$P(-5) = (-5)^3 + 4(-5)^2 - 11(-5) - 30$
$= 0$
The remainder is 0, so $x + 5$ is a factor of $x^3 + 4x^2 - 11x - 30$.

b) Determine the other binomial factors of the polynomial. Verify that the factors are correct.

Divide by $x + 5$ to determine the other factor.

-5	1	4	-11	-30
		-5	5	30
	1	-1	-6	0

So, $x^3 + 4x^2 - 11x - 30 = (x + 5)(x^2 - x - 6)$
Factor the trinomial.
$x^2 - x - 6 = (x + 2)(x - 3)$
So, $x^3 + 4x^2 - 11x - 30 = (x + 2)(x - 3)(x + 5)$
To verify, expand:
$(x + 2)(x - 3)(x + 5) = (x^2 - x - 6)(x + 5)$
$= x^3 + 5x^2 - x^2 - 5x - 6x - 30$
$= x^3 + 4x^2 - 11x - 30$
Since this is the original polynomial, the factors are correct.

11. Fully factor each polynomial.

a) $x^3 + 6x^2 + 3x - 10$

Let $P(x) = x^3 + 6x^2 + 3x - 10$
The factors of -10 are: 1, -1, 2, -2, 5, -5, 10, -10
Use mental math to substitute $x = 1$:
$P(1) = 0$
So, $x - 1$ is a factor.
Divide to determine the other factor.

1	1	6	3	-10
		1	7	10
	1	7	10	0

So, $x^3 + 6x^2 + 3x - 10 = (x - 1)(x^2 + 7x + 10)$
Factor the trinomial: $x^2 + 7x + 10 = (x + 2)(x + 5)$
So, $x^3 + 6x^2 + 3x - 10 = (x - 1)(x + 2)(x + 5)$

TEACHER NOTE

Achievement Indicator
Question 11 addresses AI 11.5: Explain and apply the factor theorem to express a polynomial expression as a product of factors.

TEACHER NOTE

Some students may use the factor theorem to determine all factors, instead of using division then factoring to determine some factors.