2. How do you decide whether to use synthetic division or the factor theorem to help you factor a polynomial?

I use synthetic division when I need to know the quotient and I use the factor theorem when I want to determine whether a binomial is a factor of a polynomial. Sometimes when I use the factor theorem, the numbers involved are not friendly, so if I don't have a calculator, I will use synthetic division. If the remainder is 0, I know the binomial is a factor.

Exercises

A

3. Write each binomial in the form $x - a$. What is the value of a?

a) $x + 4$

$x + 4 = x - (-4)$
$a = -4$

b) $x - 1$

$x - 1$ is in the form $x - a$.
$a = 1$

c) $11 + x$

$11 + x = x - (-11)$
$a = -11$

d) $-7 + x$

$-7 + x = x - 7$
$a = 7$

4. a) Determine the remainder when $x^3 - 4x^2 - 7x + 10$ is divided by each binomial.

i) $x - 1$

Let $P(x) = x^3 - 4x^2 - 7x + 10$
$P(1) = (1)^3 - 4(1)^2 - 7(1) + 10$
$= 1 - 4 - 7 + 10$
$= 0$
The remainder is 0.

ii) $x + 3$

$P(-3) = (-3)^3 - 4(-3)^2 - 7(-3) + 10$
$= -27 - 36 + 21 + 10$
$= -32$
The remainder is -32.

iii) $x + 2$

$P(-2)$
$= (-2)^3 - 4(-2)^2 - 7(-2) + 10$
$= -8 - 16 + 14 + 10$
$= 0$
The remainder is 0.

iv) $x - 2$

$P(2)$
$= (2)^3 - 4(2)^2 - 7(2) + 10$
$= 8 - 16 - 14 + 10$
$= -12$
The remainder is -12.

b) Which binomials in part a are factors of $x^3 - 4x^2 - 7x + 10$? How do you know?

$x - 1$ and $x + 2$ are factors of $x^3 - 4x^2 - 7x + 10$ because the value of the polynomial when $x = 1$ and when $x = -2$ is 0.

5. Which values of a, $a \in \mathbb{Z}$, should be chosen to test for binomial factors of the form $x - a$ of the polynomial $x^4 + 3x^3 - 8x^2 - 12x + 16$? How did you choose the values?

I chose values of a that are factors of the constant term in the polynomial, 16. Factors of 16 are: 1, −1, 2, −2, 4, −4, 8, −8, 16, −16

B

6. a) Determine the remainder when each polynomial is divided by $x - 2$.

i) $x^2 - 7x + 11$

Let $P(x) = x^2 - 7x + 11$
$P(2) = (2)^2 - 7(2) + 11$
$= 4 - 14 + 11$
$= 1$
The remainder is 1.

ii) $2x^3 - 3x^2 - 6x + 8$

Let $P(x) = 2x^3 - 3x^2 - 6x + 8$
$P(2) = 2(2)^3 - 3(2)^2 - 6(2) + 8$
$= 16 - 12 - 12 + 8$
$= 0$
The remainder is 0.

iii) $3x^3 - 2x^2 - 10x + 6$

Let $P(x) = 3x^3 - 2x^2 - 10x + 6$
$P(2) = 3(2)^3 - 2(2)^2 - 10(2) + 6$
$= 24 - 8 - 20 + 6$
$= 2$
The remainder is 2.

iv) $x^4 - 2x^3 + 3x^2 - 8$

Let $P(x) = x^4 - 2x^3 + 3x^2 - 8$
$P(2) = (2)^4 - 2(2)^3 + 3(2)^2 - 8$
$= 16 - 16 + 12 - 8$
$= 4$
The remainder is 4.

b) Explain the relationship between the remainder when a polynomial $P(x)$ is divided by $x - a$, $a \in \mathbb{Z}$, and $P(a)$.

When a polynomial $P(x)$ is divided by $x - a$, the remainder is $P(a)$.
This result comes from the division statement: $P(x) = (x - a)Q(x) + R$
When $x = a$, $x - a = 0$, so $(x - a)Q(x) = 0$
Then, $P(a) = R$

TEACHER NOTE

Achievement Indicator

Question 6b addresses AI 11.4: Explain the relationship between the remainder when a polynomial expression is divided by $x - a$, $a \in I$, and the value of the polynomial expression at $x = a$ (remainder theorem).