12. Here is a student's solution for dividing $3x^4 - 4x^3 + 5x^2 - 6x + 10$ by $x - 1$ using synthetic division. Identify the error in the solution. Write a correct solution.

$$\begin{array}{c|ccccc} -1 & 3 & -4 & 5 & -6 & 10 \\ & & -3 & 7 & -12 & 18 \\ \hline & 3 & -7 & 12 & -18 & 28 \end{array}$$

So, $3x^4 - 4x^3 + 5x^2 - 6x + 10 = (x - 1)(3x^3 - 7x^2 + 12x - 18) + 28$

The student should have used 1 instead of -1 because when comparing $x - 1$ to $x - a$, $a = 1$.
Correct solution:

$$\begin{array}{c|ccccc} 1 & 3 & -4 & 5 & -6 & 10 \\ & & 3 & -1 & 4 & -2 \\ \hline & 3 & -1 & 4 & -2 & 8 \end{array}$$

So, $3x^4 - 4x^3 + 5x^2 - 6x + 10 = (x - 1)(3x^3 - x^2 + 4x - 2) + 8$

13. A polynomial is divided by $x - a$, $a \in \mathbb{Z}$, and has quotient $2x^2 - 3x + 5$ and remainder 7. Determine a possible polynomial. How many different polynomials can you determine? Explain.

Sample response:
The division statement has the form:
Original polynomial = (binomial divisor)(quotient) + remainder
The quotient is: $2x^2 - 3x + 5$
The remainder is: 7
Choose a binomial divisor: $x + 2$
So, the original polynomial is: $(x + 2)(2x^2 - 3x + 5) + 7$

$$= 2x^3 - 3x^2 + 5x + 4x^2 - 6x + 10 + 7$$
$$= 2x^3 + x^2 - x + 17$$

I can write many different polynomials because many different binomial divisors are possible.

C

14. Divide.

$(-4x^4 + 17x^2 + 6x^5 - 27x + 10 - 21x^3) \div (x^3 - 2)$

$$\begin{array}{r} 6x^2 - 4x - 21 \\ x^3 - 2 \overline{)\,6x^5 - 4x^4 - 21x^3 + 17x^2 - 27x + 10} \\ \underline{6x^5 \qquad\quad - 12x^2} \\ -4x^4 - 21x^3 + 29x^2 - 27x \\ \underline{-4x^4 \qquad\quad + 8x} \\ -21x^3 + 29x^2 - 35x + 10 \\ \underline{-21x^3 \qquad\quad + 42} \\ 29x^2 - 35x - 32 \end{array}$$

The result is: $6x^2 - 4x - 21$ R$(29x^2 - 35x - 32)$

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