

**Check Your Understanding**

2. Which binomials are factors of  $x^3 - 6x^2 + 5x + 12$ ?
- a)  $x + 1$       b)  $x - 3$   
 c)  $x - 4$       d)  $x + 4$

Let  $P(x) = x^3 - 6x^2 + 5x + 12$

a)  $P(-1) = (-1)^3 - 6(-1)^2 + 5(-1) + 12$   
 $= 0$

Since  $P(-1) = 0$ ,  $x + 1$  is a factor.

b)  $P(3) = (3)^3 - 6(3)^2 + 5(3) + 12$   
 $= 0$

Since  $P(3) = 0$ ,  $x - 3$  is a factor.

c)  $P(4) = (4)^3 - 6(4)^2 + 5(4) + 12$   
 $= 0$

Since  $P(4) = 0$ ,  $x - 4$  is a factor.

d)  $P(-4) = (-4)^3 - 6(-4)^2 + 5(-4) + 12$   
 $= -168$

Since  $P(-4) \neq 0$ ,  $x + 4$  is not a factor.

**TEACHER NOTE**

**DI: Extending Thinking**

Ask students to determine a strategy for determining the remainder when  $x^3 + 2x^2 - 13x + 10$  is divided by  $2x - 1$ . Then have students calculate the remainder using their strategy. (Substitute  $x = 0.5$  to obtain the remainder 4.125.)

**Example 2** Determining whether a Binomial Is a Factor of a Given Polynomial

Which binomials are factors of  $x^3 + 2x^2 - 13x + 10$ ?

- a)  $x - 1$                       b)  $x - 2$   
 c)  $x + 2$                       d)  $x + 5$

**SOLUTION**

Let  $P(x) = x^3 + 2x^2 - 13x + 10$  and use the factor theorem.

a)  $P(1) = (1)^3 + 2(1)^2 - 13(1) + 10$   
 $= 0$

Since  $P(1) = 0$ ,  $x - 1$  is a factor of  $x^3 + 2x^2 - 13x + 10$ .

b)  $P(2) = (2)^3 + 2(2)^2 - 13(2) + 10$   
 $= 0$

Since  $P(2) = 0$ ,  $x - 2$  is a factor of  $x^3 + 2x^2 - 13x + 10$ .

c)  $P(-2) = (-2)^3 + 2(-2)^2 - 13(-2) + 10$   
 $= 36$

Since  $P(-2) \neq 0$ ,  $x + 2$  is not a factor of  $x^3 + 2x^2 - 13x + 10$ .

d)  $P(-5) = (-5)^3 + 2(-5)^2 - 13(-5) + 10$   
 $= 0$

Since  $P(-5) = 0$ ,  $x + 5$  is a factor of  $x^3 + 2x^2 - 13x + 10$ .

From *Example 2*, there are three binomial factors of  $x^3 + 2x^2 - 13x + 10$ :  $x - 1$ ,  $x - 2$ ,  $x + 5$

The product of these factors is the original polynomial:

$$(x - 1)(x - 2)(x + 5) = x^3 + 2x^2 - 13x + 10$$

The constant term in each binomial is a factor of the constant term in the polynomial; that is, each of 1, 2, and  $-5$  is a factor of 10.

This leads to the **factor property**.

**Factor Property**

If  $x - a$ ,  $a \in \mathbb{Z}$ , is a factor of a polynomial, then  $a$  is a factor of the constant term in the polynomial.

**Example 3** Factoring a Polynomial

Factor fully:  $2x^3 - 9x^2 + 7x + 6$

**SOLUTION**

Let  $P(x) = 2x^3 - 9x^2 + 7x + 6$  and use the factor theorem.

List the factors of the constant term, 6: 1,  $-1$ , 2,  $-2$ , 3,  $-3$ , 6,  $-6$

Use mental math to substitute  $x = 1$ , then  $x = -1$  to determine that neither  $x - 1$  nor  $x + 1$  is a factor.

Try  $x = 2$ :  $P(2) = 2(2)^3 - 9(2)^2 + 7(2) + 6$   
 $= 2(8) - 9(4) + 14 + 6$   
 $= 0$

So,  $x - 2$  is a factor of  $2x^3 - 9x^2 + 7x + 6$ .

Divide to determine the other factor.

$$\begin{array}{r|rrrr} 2 & 2 & -9 & 7 & 6 \\ & & 4 & -10 & -6 \\ \hline & 2 & -5 & -3 & 0 \end{array}$$

Since  $P(x)$  is a polynomial of degree 3, the quotient is a polynomial of degree 2.

So,  $2x^3 - 9x^2 + 7x + 6 = (x - 2)(2x^2 - 5x - 3)$

Factor the trinomial:  $2x^2 - 5x - 3 = (x - 3)(2x + 1)$

So,  $2x^3 - 9x^2 + 7x + 6 = (x - 2)(x - 3)(2x + 1)$

**THINK FURTHER**

In *Example 3*, how do you know that neither  $x - 1$  nor  $x + 1$  is a factor of  $2x^2 - 5x - 3$ ?

Since neither  $x - 1$  nor  $x + 1$  is a factor of the original polynomial, neither can be a factor of the trinomial.

**Discuss the Ideas**

1. How can you determine whether a binomial of the form  $x - a$  is a factor of a polynomial?

I can divide the polynomial by the binomial to determine the remainder. If the remainder is 0, the binomial is a factor. I can also use the factor theorem to determine the value of the polynomial when  $x = a$ . If this value is 0, the binomial is a factor.



**Check Your Understanding**

3. Factor fully:  $3x^3 - 4x^2 - 5x + 2$

Let  $P(x) = 3x^3 - 4x^2 - 5x + 2$

List the factors of the constant term, 2: 1,  $-1$ , 2,  $-2$

Use mental math to substitute  $x = 1$  to determine that  $x - 1$  is not a factor.

Try  $x = -1$ :  
 $P(-1) = 3(-1)^3 - 4(-1)^2 - 5(-1) + 2$   
 $= 0$

So,  $x + 1$  is a factor of  $3x^3 - 4x^2 - 5x + 2$ .

Divide.

$$\begin{array}{r|rrrr} -1 & 3 & -4 & -5 & 2 \\ & & -3 & 7 & -2 \\ \hline & 3 & -7 & 2 & 0 \end{array}$$

So,  $3x^3 - 4x^2 - 5x + 2 = (x + 1)(3x^2 - 7x + 2)$

Factor the trinomial.

$3x^2 - 7x + 2 = (x - 2)(3x - 1)$

So,  $3x^3 - 4x^2 - 5x + 2 = (x + 1)(x - 2)(3x - 1)$

**TEACHER NOTE**

**DI: Common Difficulties**

Students who are having difficulty factoring trinomials could complete Master 1.1a Activate Prior Learning: Factoring Polynomials to review the strategies involved.

**TEACHER NOTE**

**DI: Common Difficulties**

In *Example 3*, remind students that the binomial factors can be written in any order, and that the presentation of the solution may differ depending on which binomial factor is determined first.