Example 2 Determining whether a Binomial Is a Factor of a Given Polynomial

Which binomials are factors of $x^3 + 2x^2 - 13x + 10$?

a) $x - 1$ **b)** $x - 2$

c) $x + 2$ **d)** $x + 5$

SOLUTION

Let $P(x) = x^3 + 2x^2 - 13x + 10$ and use the factor theorem.

a) $P(1) = (1)^3 + 2(1)^2 - 13(1) + 10$
$= 0$

Since $P(1) = 0$, $x - 1$ is a factor of $x^3 + 2x^2 - 13x + 10$.

b) $P(2) = (2)^3 + 2(2)^2 - 13(2) + 10$
$= 0$

Since $P(2) = 0$, $x - 2$ is a factor of $x^3 + 2x^2 - 13x + 10$.

c) $P(-2) = (-2)^3 + 2(-2)^2 - 13(-2) + 10$
$= 36$

Since $P(-2) \neq 0$, $x + 2$ is not a factor of $x^3 + 2x^2 - 13x + 10$.

d) $P(-5) = (-5)^3 + 2(-5)^2 - 13(-5) + 10$
$= 0$

Since $P(-5) = 0$, $x + 5$ is a factor of $x^3 + 2x^2 - 13x + 10$.

From *Example 2*, there are three binomial factors of $x^3 + 2x^2 - 13x + 10$:
$x - 1, x - 2, x + 5$

The product of these factors is the original polynomial:
$(x - \mathbf{1})(x - \mathbf{2})(x + \mathbf{5}) = x^3 + 2x^2 - 13x + \mathbf{10}$

The constant term in each binomial is a factor of the constant term in the polynomial; that is, each of 1, 2, and −5 is a factor of 10.

This leads to the **factor property**.

Factor Property

If $x - a$, $a \in \mathbb{Z}$, is a factor of a polynomial, then a is a factor of the constant term in the polynomial.

Check Your Understanding

2. Which binomials are factors of $x^3 - 6x^2 + 5x + 12$?

a) $x + 1$ b) $x - 3$

c) $x - 4$ d) $x + 4$

Let $P(x) = x^3 - 6x^2 + 5x + 12$

a) $P(-1) = (-1)^3 - 6(-1)^2 + 5(-1) + 12$
$= 0$

Since $P(-1) = 0$, $x + 1$ is a factor.

b) $P(3) = (3)^3 - 6(3)^2 + 5(3) + 12$
$= 0$

Since $P(3) = 0$, $x - 3$ is a factor.

c) $P(4) = (4)^3 - 6(4)^2 + 5(4) + 12$
$= 0$

Since $P(4) = 0$, $x - 4$ is a factor.

d) $P(-4) = (-4)^3 - 6(-4)^2 + 5(-4) + 12$
$= -168$

Since $P(-4) \neq 0$, $x + 4$ is not a factor.

TEACHER NOTE

DI: Extending Thinking

Ask students to determine a strategy for determining the remainder when $x^3 + 2x^2 - 13x + 10$ is divided by $2x - 1$.
Then have students calculate the remainder using their strategy. (*Substitute $x = 0.5$ to obtain the remainder 4.125.*)

Example 3 Factoring a Polynomial

Factor fully: $2x^3 - 9x^2 + 7x + 6$

SOLUTION

Let $P(x) = 2x^3 - 9x^2 + 7x + 6$ and use the factor theorem.
List the factors of the constant term, 6: 1, −1, 2, −2, 3, −3, 6, −6
Use mental math to substitute $x = 1$, then $x = -1$ to determine that neither $x - 1$ nor $x + 1$ is a factor.
Try $x = 2$: $P(2) = 2(2)^3 - 9(2)^2 + 7(2) + 6$
$= 2(8) - 9(4) + 14 + 6$
$= 0$
So, $x - 2$ is a factor of $2x^3 - 9x^2 + 7x + 6$.
Divide to determine the other factor.

2	2	−9	7	6
		4	−10	−6
	2	−5	−3	0

Since $P(x)$ is a polynomial of degree 3, the quotient is a polynomial of degree 2.
So, $2x^3 - 9x^2 + 7x + 6 = (x - 2)(2x^2 - 5x - 3)$
Factor the trinomial: $2x^2 - 5x - 3 = (x - 3)(2x + 1)$
So, $2x^3 - 9x^2 + 7x + 6 = (x - 2)(x - 3)(2x + 1)$

Animation

Check Your Understanding

3. Factor fully: $3x^3 - 4x^2 - 5x + 2$

Let $P(x) = 3x^3 - 4x^2 - 5x + 2$
List the factors of the constant term, 2: 1, −1, 2, −2
Use mental math to substitute $x = 1$ to determine that $x - 1$ is not a factor.
Try $x = -1$:
$P(-1) = 3(-1)^3 - 4(-1)^2 - 5(-1) + 2$
$= 0$
So, $x + 1$ is a factor of $3x^3 - 4x^2 - 5x + 2$.
Divide.

−1	3	−4	−5	2
		−3	7	−2
	3	−7	2	0

So, $3x^3 - 4x^2 - 5x + 2 = (x + 1)(3x^2 - 7x + 2)$
Factor the trinomial.
$3x^2 - 7x + 2 = (x - 2)(3x - 1)$
So, $3x^3 - 4x^2 - 5x + 2 = (x + 1)(x - 2)(3x - 1)$

THINK FURTHER

In *Example 3*, how do you know that neither $x - 1$ nor $x + 1$ is a factor of $2x^2 - 5x - 3$?

Since neither $x - 1$ nor $x + 1$ is a factor of the original polynomial, neither can be a factor of the trinomial.

TEACHER NOTE

DI: Common Difficulties

Students who are having difficulty factoring trinomials could complete Master 1.1a Activate Prior Learning: Factoring Polynomials to review the strategies involved.

Discuss the Ideas

1. How can you determine whether a binomial of the form $x - a$ is a factor of a polynomial?

I can divide the polynomial by the binomial to determine the remainder. If the remainder is 0, the binomial is a factor. I can also use the factor theorem to determine the value of the polynomial when $x = a$. If this value is 0, the binomial is a factor.

TEACHER NOTE

DI: Common Difficulties

In *Example 3*, remind students that the binomial factors can be written in any order, and that the presentation of the solution may differ depending on which binomial factor is determined first.