Example 1 Determining the Remainder without Dividing

Determine the remainder when $3x^4 + 7x^3 - x^2 + 14x - 3$ is divided by each binomial.

a) $x - 1$ **b)** $x + 3$

SOLUTION

Let $P(x) = 3x^4 + 7x^3 - x^2 + 14x - 3$ and use the remainder theorem.

a) When $P(x)$ is divided by $x - 1$, the remainder is $P(1)$.

$$\begin{aligned} P(1) &= 3(1)^4 + 7(1)^3 - (1)^2 + 14(1) - 3 \\ &= 3 + 7 - 1 + 14 - 3 \\ &= 20 \end{aligned}$$

The remainder is 20.

b) Write $x + 3$ in the form $x - a$: $x - (-3)$

When $P(x)$ is divided by $x + 3$, the remainder is $P(-3)$.

$$\begin{aligned} P(-3) &= 3(-3)^4 + 7(-3)^3 - (-3)^2 + 14(-3) - 3 \\ &= 3(81) + 7(-27) - 9 - 42 - 3 \\ &= 243 - 189 - 9 - 42 - 3 \\ &= 0 \end{aligned}$$

The remainder is 0.

Check Your Understanding

1. Determine the remainder when $2x^4 - 5x^3 - 5x^2 + 5x + 3$ is divided by each binomial.

a) $x - 3$ b) $x + 2$

Let $P(x) = 2x^4 - 5x^3 - 5x^2 + 5x + 3$

a) When $P(x)$ is divided by $x - 3$, the remainder is $P(3)$.

$$\begin{aligned} P(3) &= 2(3)^4 - 5(3)^3 - 5(3)^2 + 5(3) + 3 \\ &= 2(81) - 5(27) - 5(9) + 15 + 3 \\ &= 0 \end{aligned}$$

The remainder is 0.

b) When $P(x)$ is divided by $x + 2$, the remainder is $P(-2)$.

$$\begin{aligned} P(-2) &= 2(-2)^4 - 5(-2)^3 - 5(-2)^2 + 5(-2) + 3 \\ &= 2(16) - 5(-8) - 5(4) - 10 + 3 \\ &= 45 \end{aligned}$$

The remainder is 45.

THINK FURTHER

Does a polynomial have to be written in descending order to use the remainder theorem?

No, I evaluate each term, then add. Since addition is commutative, order does not matter.

In *Example 1b*, the remainder was 0. This means that the divisor, $x + 3$, is a factor of the polynomial. This special case of the remainder theorem is called the **factor theorem.**

Factor Theorem

For $a \in \mathbb{Z}$, $x - a$ is a factor of the polynomial $P(x)$ if $P(a) = 0$.

TEACHER NOTE

Students could use long division or synthetic division to verify that their remainders are correct.