$$
\begin{array}{r}
2x^3 - 5x^2 - 7x + 3 \\
x + 2\overline{)2x^4 - x^3 - 17x^2 - 11x + 6} \\
\underline{2x^4 + 4x^3} \qquad\qquad\qquad\qquad \\
-5x^3 - 17x^2 \qquad\qquad\quad \\
\underline{-5x^3 - 10x^2} \qquad\qquad\quad \\
-7x^2 - 11x \qquad \\
\underline{-7x^2 - 14x} \qquad \\
3x + 6 \\
\underline{3x + 6} \\
0
\end{array}
$$

The remainder is 0, so $x + 2$ is a factor.

$P(-2) = 2(16) - (-8) - 17(4) + 22 + 6$
$= 0$

A binomial of the form $x - a$ is a factor of a polynomial if the value of the polynomial is 0 when $x = a$.

TEACHER NOTE

Have students insert subtraction signs and brackets if they need them to understand the long division process.

When a polynomial is divided by a binomial of the form $x - a$, the remainder is related to the polynomial and the constant term of the divisor. For example,

$$
\begin{array}{r}
5x^2 + 8x + 24 \quad \\
x - 2\overline{)5x^3 - 2x^2 + 8x - 1} \\
\underline{5x^3 - 10x^2} \qquad\qquad\quad \\
8x^2 + 8x \qquad \\
\underline{8x^2 - 16x} \qquad \\
24x - 1 \\
\underline{24x - 48} \\
47
\end{array}
$$

The divisor is $x - 2$.
Let $P(x) = 5x^3 - 2x^2 + 8x - 1$

Evaluate the polynomial for $x = 2$.

$P(2) = 5(2)^3 - 2(2)^2 + 8(2) - 1$
$= 40 - 8 + 16 - 1$
$= 47$

The value of the polynomial when $x = 2$ is equal to the remainder when the polynomial is divided by $x - 2$. This is called the **remainder theorem**.

Remainder Theorem

When a polynomial, $P(x)$, is divided by a binomial, $x - a$, $a \in \mathbb{Z}$, the remainder is $P(a)$.

To illustrate why this works, use the division statement.

$P(x) = (x - a)Q(x) + R$
Substitute: $x = a$
$P(a) = (a - a)Q(a) + R$
$P(a) = (0)Q(a) + R$
$P(a) = R$