CHAPTER 1: Polynomial Functions Examples Booklet

40S Pre-Calculus

Winnipeg Adult Education Centre
Fall 2018

Unit Lessons

Lesson 1: Factoring Review

Lesson 2: Dividing a Polynomial by a Binomial

Lesson 3: Factoring Polynomials

Lesson 4: Relating Polynomial Functions and Equations

Lesson 5: Sketching the Graph of a Polynomial Function

Lesson 6: Modelling and Solving Problems with Polynomial Functions

Homework:

Most assignments for practice will be from your workbook: Pearson Pre-Calculus 12. Other assignments will be handed out for marks as needed.

Learning Outcomes

At the end of this unit, you should be able to:

Demonstrate an understanding of factoring polynomials of degree greater then 2:

- Explain how long division of a polynomial expression is related to synthetic division.
- Divide a polynomial expression by a binomial expression using long division or synthetic division.
- Explain the relationship between the factors of a polynomial expression and the zeros of the corresponding polynomial function.
- Explain and apply the remainder theorem to solve problems.
- Explain and apply the factor theorem to express a polynomial expression as a product of its factors.

Graph and analyze polynomial functions:

- Identify polynomial functions from a given set of functions.
- Explain the role of the constant term and the leading coefficient of a polynomial function and their relationships to its graph.
- Know how an odd or even degree affects the graph of a polynomial function.
- Explain how the multiplicity of a zero affects the graph of a polynomial function.
- Sketch the graph of a polynomial function.
- Model and solve problems involving polynomial functions.

Key Terms/Vocabulary

Dividend

Divisor

Quotient

Quadratic Function

Cubic Function

Quartic Function

Quintic Function

Remainder Theorem

Factor Theorem

Synthetic Division

Leading Coefficient

Constant Term

Odd Degree

Even Degree

End Behaviour

Multiplicity

Lesson 1: Factoring Review

In previous courses, you learned four main types of factoring binomials and trinomials:

1. Common factoring

a) $3xy + 12x^4y^7$

b) $5m^2n + 25m^2n^2 - 45m^4n^5$

2. Difference of Squares

a) $x^2 - 100$

b) $64x^2 - 25y^2$

c) $4 - 49m^2$

3. Simple Trinomials

a) $x^2 + 7x + 12$

b) $x^2 + 4x - 45$

c) $v^2 - 2v - 63$

4. Complex Trinomials

a) $5x^2 + 9x + 4$

b) $4x^2 - 7x + 3$

c) $6x^2 + x - 2$

Lesson 2: Dividing a Polynomial by a Binomial

Example 1: Divide $x^3 \mp 2x^2 - 5x - 6$ by $x - 2$. **Verify** your answer.

Example 2: Divide $3x + 4x^2 - 2 + x^3$ by $x + 2$

Example 3: Divide $5x + 3x^3 - 8$ by $x + 3$

Example 4: Divide $x^4 - x^3 - 6x^2 + 4x + 8$ by $x + 1$.

Use synthetic division to divide:

Example 5: $(x^3 - x^2 - 5x + 6) \div (x - 2)$

Example 6: $(2x^4 - 3x^2 + 4x - 2) \div (x + 1)$

Example 7: A polynomial is divided by $x - 2$. The quotient is $x^2 + x - 3$. What is the original polynomial?

Lesson 3: Factoring Polynomials

Remainder Theorem: When a polynomial is divided by a binomial of the form $x - a$, the remainder is $P(a)$. This means when we replace "x" with the value of "a" in the polynomial, the number we end up with is the remainder.

Example 1: Determine the remainder when $2x^4 - 5x^3 - 5x^2 + 5x + 3$ is divided by

a) $x - 3$

b) $x + 2$

c) $x + 1$

When we end up with a remainder of 0, we have found a factor of the polynomial.

Factor Theorem: $x - a$ is a factor of the polynomial $P(x)$ if $P(a) = 0$.

Also, if $x - a$ is a factor of a polynomial, then a is a factor of the constant term of the polynomial.

Example 2: Use the factor theorem to determine which of the following binomials are factors of the polynomial $x^3 - 6x^2 + 5x + 12$

a) $x + 1$

b) $x - 3$

c) $x - 4$

d) $x + 4$

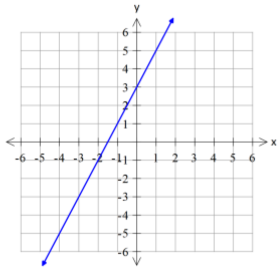
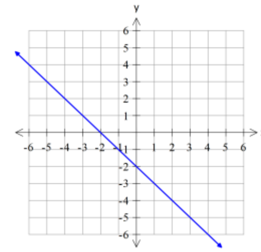
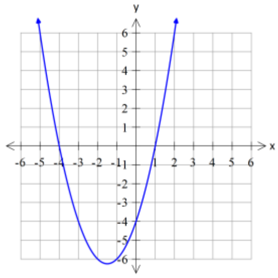
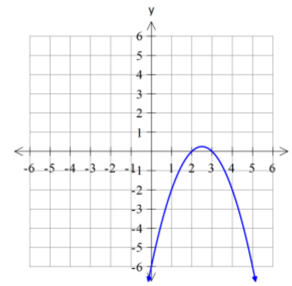
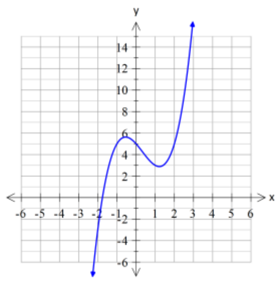
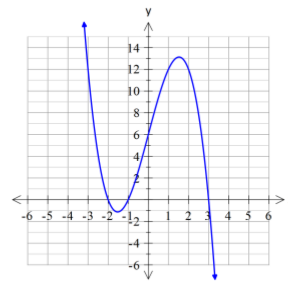
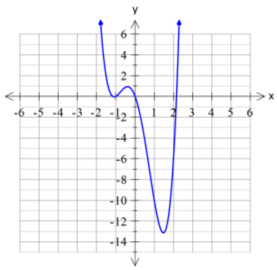
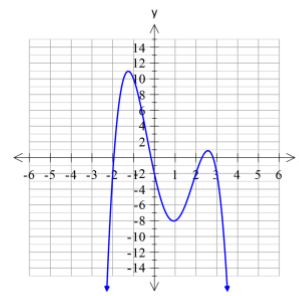
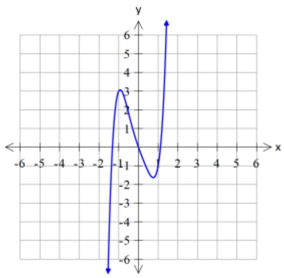
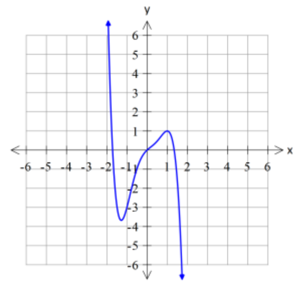
Example 3: Determine one binomial factor of the polynomial $x^3 - x^2 - 8x + 12$

Example 4: Factor fully: $3x^3 - 4x^2 - 5x + 2$

Example 5: What value of "k" will ensure that $x + 2$ is a factor of $x^3 - kx^2 + 3x - 5$?

Lesson 4: Relating Polynomial Functions and Equations

Inspect each of the following polynomial functions. Can you see any patterns?

a) $y = 2x + 3$ 	b) $y = -x - 2$ 
c) $y = x^2 + 3x - 4$ 	d) $y = -x^2 + 5x - 6$ 
e) $y = x^3 - x^2 - 2x + 5$ 	f) $y = -x^3 + 7x + 6$ 
g) $y = 2x^4 - 7x^2 - 5x$ 	h) $y = -x^4 + 3x^3 + 4x^2 -$ $12x - 2$ 
i) $y = 2x^5 + x^3 - x^2 +$ $3x$ 	j) $y = -x^5 + 2x^3 - x^2 +$ x 

After inspecting the graphs in the chart on the previous page, respond to the following questions.

a) How does the sign of the leading coefficient affect the shape of the graph?

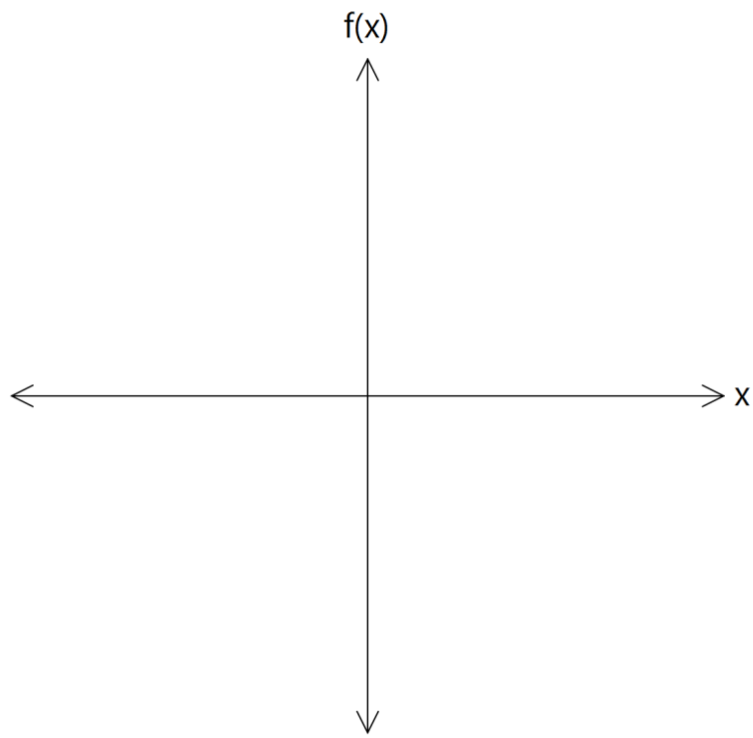
b) What does the value of the constant term tell us about the graph?

c) What characteristics do the odd-degree functions share?

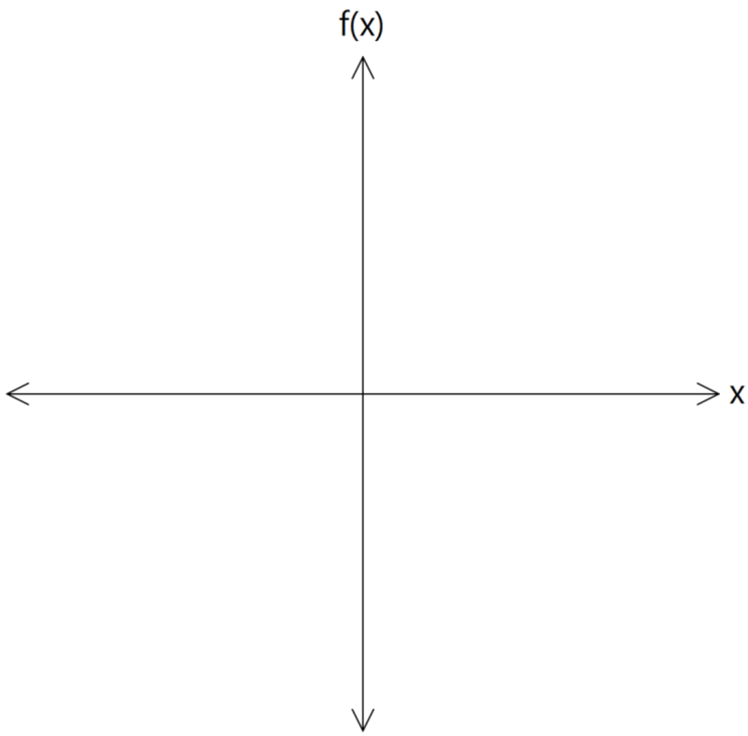
d) what characteristics do the even-degree functions share?

Lesson 5: Sketching the Graph of a Polynomial Function

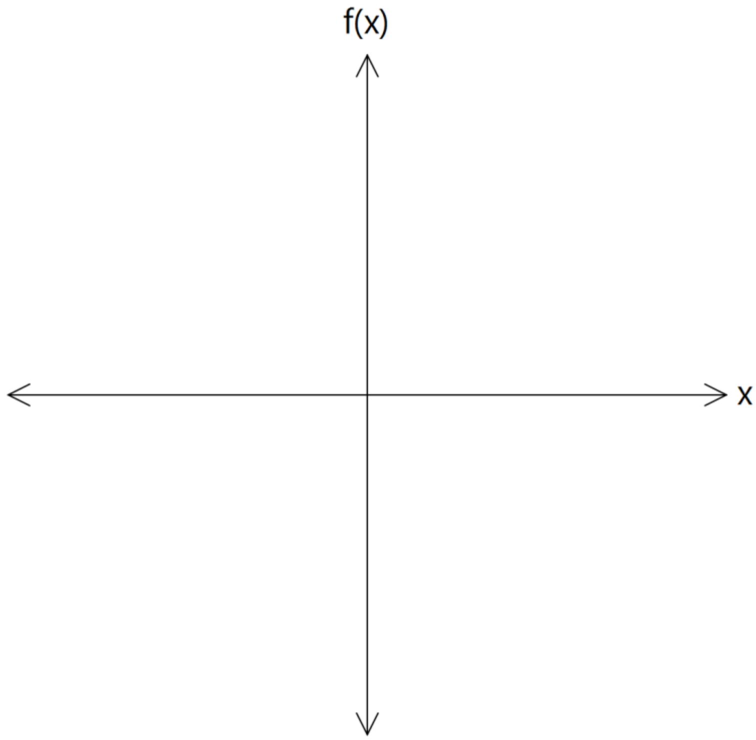
Example 1: Sketch the graph of $f(x) = x^4 + 2x^3 - 7x^2 - 8x + 12$ using intercepts and your knowledge of polynomial functions.



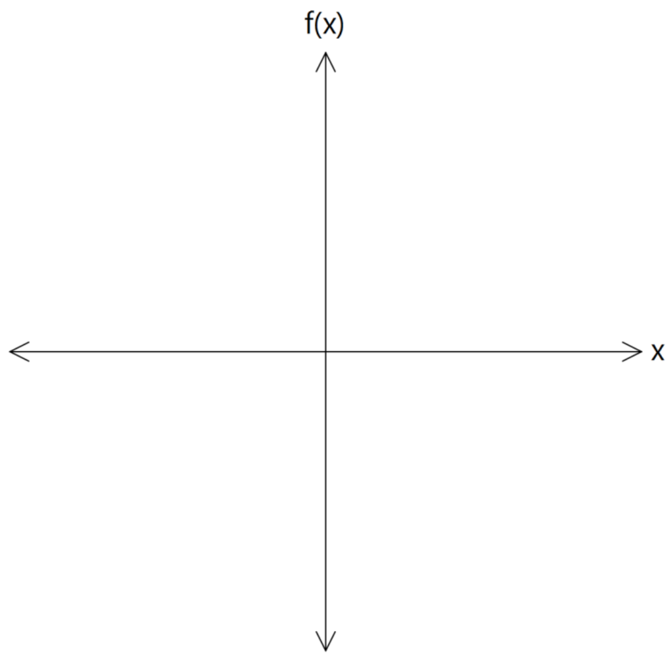
Example 2: Sketch the graph of $f(x) = (x-1)^2(x+3)^2$ using the multiplicity of zeros.



Example 3: Sketch the graph of $f(x) = -(x+1)^3(x-2)^2$



Example 4: Sketch a possible graph of a cubic function with: a negative leading coefficient; a zero of 5 of multiplicity 2; a zero of 3 of multiplicity 1.



Example 5: Write a possible equation for a quartic function with a zero of 3 of multiplicity 2 and a zero of 1 of multiplicity 2.

Lesson 6: Modelling and Solving Problems with Polynomial Functions

Example 1: The volume, in cubic centimetres, of a rectangular box can be modeled by the polynomial function $V(x) = 3x^3 + x^2 - 12x - 4$. Determine expressions for the other dimensions of the box if the height is $x + 2$.

Example 2: The product of four integers is $x^4 + 6x^3 + 11x^2 + 6x$ where x is one of the integers. What are possible expressions for the other three integers?