

Lesson 7.2 Exercises, pages 537–543

A

3. Simplify each expression.

a) $\frac{3}{4} \cdot \frac{2b}{5}$

$$\begin{aligned} &= \frac{\cancel{3}^3}{\cancel{4}_2} \cdot \frac{\cancel{2}^2 b}{5} \\ &= \frac{3b}{10} \end{aligned}$$

b) $\frac{p^2}{5} \cdot \frac{15}{p}$

Non-permissible value: $p = 0$

$$\begin{aligned} \frac{p^2}{5} \cdot \frac{15}{p} &= \frac{\cancel{p}^2}{5} \cdot \frac{\cancel{15}^3}{\cancel{p}^1} \\ &= 3p, p \neq 0 \end{aligned}$$

c) $\frac{-4x}{3} \cdot \frac{9x}{2}$

$$\begin{aligned} &= \frac{-\cancel{4}^2 x}{\cancel{3}} \cdot \frac{\cancel{9}^3 x}{2} \\ &= -6x^2 \end{aligned}$$

d) $\frac{c^3}{10} \cdot \frac{-5}{2c}$

Non-permissible value: $c = 0$

$$\begin{aligned} \frac{c^3}{10} \cdot \frac{-5}{2c} &= \frac{\cancel{c}^3}{\cancel{10}_2} \cdot \frac{-\cancel{5}}{\cancel{2}^1 c} \\ &= \frac{-c^2}{4}, c \neq 0 \end{aligned}$$

4. Simplify each expression.

a) $\frac{2}{n} \div \frac{4}{n}$

Non-permissible value: $n = 0$

$$\begin{aligned} \frac{2}{n} \div \frac{4}{n} &= \frac{2}{\cancel{n}} \cdot \frac{\cancel{n}}{4} \\ &= \frac{1}{2}, n \neq 0 \end{aligned}$$

b) $\frac{2b}{5} \div \frac{4}{10b}$

Non-permissible value: $b = 0$

$$\begin{aligned} \frac{2b}{5} \div \frac{4}{10b} &= \frac{2b}{5} \cdot \frac{10b}{4} \\ &= b^2, b \neq 0 \end{aligned}$$

c) $\frac{6}{x} \div 3$

Non-permissible value: $x = 0$

$$\begin{aligned} \frac{6}{x} \div 3 &= \frac{2\cancel{3}}{x} \cdot \frac{1}{\cancel{3}} \\ &= \frac{2}{x}, x \neq 0 \end{aligned}$$

d) $\frac{d^3}{4} \div \frac{d}{12}$

Non-permissible value: $d = 0$

$$\begin{aligned} \frac{d^3}{4} \div \frac{d}{12} &= \frac{d^{\cancel{3}2}}{4} \cdot \frac{12^{\cancel{3}}}{\cancel{d}} \\ &= 3d^2, d \neq 0 \end{aligned}$$

5. Simplify each expression.

a) $\frac{m}{5p} \cdot \frac{10p^2}{3m}$

Non-permissible values:

$p = 0$ and $m = 0$

$$\begin{aligned} \frac{m}{5p} \cdot \frac{10p^2}{3m} &= \frac{\cancel{m}}{5\cancel{p}} \cdot \frac{2\cancel{10}p^{\cancel{2}}}{3\cancel{m}} \\ &= \frac{2p}{3}, p \neq 0, m \neq 0 \end{aligned}$$

b) $\frac{-2ab^2}{5a^3b} \div \frac{-4a}{15ab}$

Non-permissible values:

$a = 0$ and $b = 0$

$$\begin{aligned} \frac{-2ab^2}{5a^3b} \div \frac{-4a}{15ab} &= \frac{\cancel{-2}ab^{\cancel{2}}}{5\cancel{a}^{\cancel{3}2}b} \cdot \frac{3\cancel{15}ab}{\cancel{2}4a} \\ &= \frac{3b^2}{2a^2}, a \neq 0, b \neq 0 \end{aligned}$$

B

6. Simplify each expression.

a) $\frac{3x}{2(x-3)} \cdot \frac{8(x-3)}{9x^2}$

Non-permissible values:

$x = 0$ and $x = 3$

$$\begin{aligned} &= \frac{\cancel{3}x}{\cancel{2}(x-\cancel{3})} \cdot \frac{4\cancel{8}(x-\cancel{3})}{\cancel{3}9x^{\cancel{2}}} \\ &= \frac{4}{3x}, x \neq 0, 3 \end{aligned}$$

b) $\frac{15(x+5)}{2x^2} \cdot \frac{8x}{5(x+5)^2}$

Non-permissible values:

$x = 0$ and $x = -5$

$$\begin{aligned} &= \frac{3\cancel{15}(x+\cancel{5})}{\cancel{2}x^{\cancel{2}}} \cdot \frac{4\cancel{8}x}{\cancel{5}(x+5)^{\cancel{2}}} \\ &= \frac{12}{x(x+5)}, x \neq -5, 0 \end{aligned}$$

$$\text{c) } \frac{2(x-4)(x+5)}{3(x+1)} \div \frac{4(x+5)}{(x+1)^2}$$

Non-permissible values:

$$x = -5 \text{ and } x = -1$$

$$\begin{aligned} &= \frac{\cancel{2}(x-4)\cancel{(x+5)}}{3(x+1)} \cdot \frac{(x+1)^2}{\cancel{4}\cancel{(x+5)}} \\ &= \frac{(x-4)(x+1)}{6}, x \neq -5, -1 \end{aligned}$$

$$\text{d) } \frac{10y}{(y-3)^3} \div \frac{y(y+1)}{(y-3)^2}$$

Non-permissible values:

$$y = 3, y = 0, \text{ and } y = -1$$

$$\begin{aligned} &= \frac{10y}{(y-3)^3} \cdot \frac{\cancel{(y-3)}^2}{y(y+1)} \\ &= \frac{10}{(y-3)(y+1)}, y \neq -1, 0, 3 \end{aligned}$$

7. Simplify each expression.

$$\text{a) } \frac{2a+2b}{3a-6b} \cdot \frac{a-2b}{a+b}$$

Non-permissible values:

$$a = 2b \text{ and } a = -b$$

$$\begin{aligned} &\frac{2a+2b}{3a-6b} \cdot \frac{a-2b}{a+b} \\ &= \frac{2\cancel{(a+b)}}{3\cancel{(a-2b)}} \cdot \frac{\cancel{a-2b}}{\cancel{a+b}} \\ &= \frac{2}{3}, a \neq 2b, -b \end{aligned}$$

$$\text{b) } \frac{n^2-4}{2n+1} \cdot \frac{6n+3}{n-2}$$

Non-permissible values:

$$n = -\frac{1}{2} \text{ and } n = 2$$

$$\begin{aligned} &\frac{n^2-4}{2n+1} \cdot \frac{6n+3}{n-2} \\ &= \frac{\cancel{(n-2)}(n+2)}{2n+1} \cdot \frac{3\cancel{(2n+1)}}{\cancel{n-2}} \\ &= 3(n+2), n \neq -\frac{1}{2}, 2 \end{aligned}$$

$$\text{c) } \frac{x+5}{x-4} \div \frac{x^2-25}{3x-12}$$

$$= \frac{x+5}{x-4} \div \frac{(x-5)(x+5)}{3(x-4)}$$

Non-permissible values:

$$x = -5, x = 4, \text{ and } x = 5$$

$$\begin{aligned} &= \frac{\cancel{x+5}}{\cancel{x-4}} \cdot \frac{3\cancel{(x-4)}}{(x-5)\cancel{(x+5)}} \\ &= \frac{3}{x-5}, x \neq -5, 4, 5 \end{aligned}$$

$$\text{d) } \frac{(b+2)^2}{3b^2-3} \div \frac{2b+4}{1-b}$$

$$= \frac{(b+2)^2}{3(b^2-1)} \div \frac{2(b+2)}{1-b}$$

$$= \frac{(b+2)^2}{3(b-1)(b+1)} \div \frac{2(b+2)}{-(b-1)}$$

Non-permissible values:

$$b = -2, b = -1, \text{ and } b = 1$$

$$\begin{aligned} &= \frac{(b+2)^2}{3\cancel{(b-1)}(b+1)} \cdot \frac{\cancel{-(b-1)}}{2(b+2)} \\ &= -\frac{b+2}{6(b+1)}, b \neq -2, -1, 1 \end{aligned}$$

8. Simplify each expression.

$$\text{a) } \frac{x^2-x-6}{x+4} \cdot \frac{x^2-16}{x^2+2x}$$

$$= \frac{(x-3)(x+2)}{x+4} \cdot \frac{(x-4)(x+4)}{x(x+2)}$$

Non-permissible values:

$$x = -4, -2, 0$$

$$\begin{aligned} &= \frac{\cancel{(x-3)}\cancel{(x+2)}}{\cancel{x+4}} \cdot \frac{(x-4)\cancel{(x+4)}}{x\cancel{(x+2)}} \\ &= \frac{(x-3)(x-4)}{x}, x \neq -4, -2, 0 \end{aligned}$$

$$\text{b) } \frac{2x^2-x-1}{x^2+2x-3} \cdot \frac{4x^2+28x+48}{2x^2-13x-7}$$

$$= \frac{(2x+1)(x-1)}{(x+3)(x-1)} \cdot \frac{4(x^2+7x+12)}{(2x+1)(x-7)}$$

$$= \frac{(2x+1)(x-1)}{(x+3)(x-1)} \cdot \frac{4(x+3)(x+4)}{(2x+1)(x-7)}$$

Non-permissible values:

$$x = -3, -\frac{1}{2}, 1, 7$$

$$\begin{aligned} &= \frac{\cancel{(2x+1)}\cancel{(x-1)}}{\cancel{(x+3)}\cancel{(x-1)}} \cdot \frac{4\cancel{(x+3)}(x+4)}{\cancel{(2x+1)}(x-7)} \\ &= \frac{4(x+4)}{x-7}, x \neq -3, -\frac{1}{2}, 1, 7 \end{aligned}$$

$$\begin{aligned} \text{c) } \frac{m^2 - m - 12}{m + 3} \div \frac{3m - 12}{m^2 - 9} \\ = \frac{(m - 4)(m + 3)}{m + 3} \div \frac{3(m - 4)}{(m - 3)(m + 3)} \end{aligned}$$

Non-permissible values:

$$m = -3, 3, 4$$

$$\begin{aligned} = \frac{\cancel{(m - 4)} \cancel{(m + 3)}}{m + 3} \cdot \frac{(m - 3)(m + 3)}{3 \cancel{(m - 4)}} \\ = \frac{(m - 3)(m + 3)}{3}, m \neq -3, 3, 4 \end{aligned}$$

$$\begin{aligned} \text{d) } \frac{2s^2 - 7s - 4}{6s^2 - 5s - 6} \div \frac{4s^2 + 4s + 1}{12s + 8} \\ = \frac{(2s + 1)(s - 4)}{(3s + 2)(2s - 3)} \div \frac{(2s + 1)(2s + 1)}{4(3s + 2)} \end{aligned}$$

Non-permissible values:

$$s = -\frac{2}{3}, -\frac{1}{2}, \frac{3}{2}$$

$$\begin{aligned} = \frac{\cancel{(2s + 1)}(s - 4)}{\cancel{(3s + 2)}(2s - 3)} \cdot \frac{4 \cancel{(3s + 2)}}{(2s + 1) \cancel{(2s + 1)}} \\ = \frac{4(s - 4)}{(2s - 3)(2s + 1)}, s \neq -\frac{2}{3}, -\frac{1}{2}, \frac{3}{2} \end{aligned}$$

9. Each rational expression is the product of two different rational expressions. What might the rational expressions have been?

Describe the strategy you used to find out.

a) $\frac{3}{y}$

b) $\frac{x + 1}{3x}$

I wrote two polynomial expressions such that when I divided the numerators and denominators by their common factors, I was left with the given product. I then expanded where necessary.

Sample Response:

$$\frac{3}{y} = \frac{3(2x)}{(5)} \cdot \frac{(5)}{(2x)y}$$

$$= \frac{6x}{5} \cdot \frac{5}{2xy}, x \neq 0, y \neq 0$$

$$\frac{x + 1}{3x} = \frac{(x + 1)(x + 2)}{3x(2)} \cdot \frac{(2)}{(x + 2)}$$

$$= \frac{x^2 + 3x + 2}{6x} \cdot \frac{2}{x + 2}, x \neq -2, 0$$

10. Each rational expression is the quotient of two different rational expressions. What might the rational expressions have been?

How do you know?

a) $\frac{3}{x - 5}$

b) $\frac{x - 9}{x + 8}$

I wrote a product of two polynomial expressions such that when I divided the numerators and denominators by their common factors, I was left with the given expression. I then expanded where necessary, inverted the second expression, then changed multiplication to division.

Sample Response:

$$\frac{3}{x - 5} = \frac{3(5)}{(x - 5)(x + 4)} \cdot \frac{x + 4}{(5)}$$

$$= \frac{15}{(x - 5)(x + 4)} \div \frac{5}{x + 4}$$

$$x \neq -4, 5$$

$$\frac{x - 9}{x + 8} = \frac{(x - 9)^2}{2(x + 8)} \cdot \frac{2}{(x - 9)}$$

$$= \frac{x^2 - 18x + 81}{2x + 16} \div \frac{x - 9}{2}$$

$$x \neq -8, 9$$

11. Simplify each expression.

$$\text{a) } \frac{3x^3y}{18x^3} \cdot \frac{2y^2}{4xy^5} \cdot \frac{8x^3y^2}{12xy}$$

Non-permissible values:

$$x = 0, y = 0$$

$$\begin{aligned} &= \frac{\cancel{3}x^{\cancel{3}}y}{\cancel{3}\cancel{18}x^{\cancel{3}}} \cdot \frac{\cancel{2}y^{\cancel{2}}}{\cancel{4}xy^{\cancel{5}}} \cdot \frac{\cancel{2}8x^{\cancel{3}}y^{\cancel{2}}}{\cancel{6}\cancel{12}xy} \\ &= \frac{x}{18y}, x \neq 0, y \neq 0 \end{aligned}$$

$$\text{b) } \frac{3ab}{4ab^3} \cdot \frac{8b^2}{27a^5b^3} \div \frac{2a^2b^2}{6ab^3}$$

Non-permissible values:

$$a = 0, b = 0$$

$$\begin{aligned} &= \frac{\cancel{3}a\cancel{b}}{\cancel{4}a\cancel{b}^3} \cdot \frac{\cancel{2}8b^{\cancel{2}}}{\cancel{3}\cancel{27}a^5b^{\cancel{3}}} \cdot \frac{\cancel{2}8a^{\cancel{2}}b^{\cancel{2}}}{\cancel{2}a^{\cancel{2}}b^{\cancel{2}}} \\ &= \frac{2}{3a^6b^2}, a \neq 0, b \neq 0 \end{aligned}$$

12. Simplify each expression.

$$\text{a) } \frac{x+3}{x^2-2x-15} \cdot \frac{2x-10}{2x^2+15x+7} \div \frac{2x-4}{x^2+5x-14}$$

$$= \frac{x+3}{(x-5)(x+3)} \cdot \frac{2(x-5)}{(2x+1)(x+7)} \div \frac{2(x-2)}{(x+7)(x-2)}$$

Non-permissible values: $x = -7, -3, -\frac{1}{2}, 2, 5$

$$\begin{aligned} &= \frac{\cancel{x+3}}{\cancel{(x-5)}(x+3)} \cdot \frac{\cancel{2}(x-5)}{(2x+1)\cancel{(x+7)}} \cdot \frac{\cancel{(x+7)}\cancel{(x-2)}}{\cancel{2}(x-2)} \\ &= \frac{1}{2x+1}, x \neq -7, -3, -\frac{1}{2}, 2, 5 \end{aligned}$$

$$\text{b) } \frac{x^2-x-56}{x+8} \div \frac{x^2+14x+49}{x^2-6x-16} \cdot \frac{3x+24}{x^2-16x+64}$$

$$= \frac{(x-8)(x+7)}{x+8} \div \frac{(x+7)(x+7)}{(x-8)(x+2)} \cdot \frac{3(x+8)}{(x-8)(x-8)}$$

Non-permissible values: $x = -8, -7, -2, 8$

$$\begin{aligned} &= \frac{\cancel{(x-8)}(x+7)}{x+8} \cdot \frac{\cancel{(x+7)}(x+2)}{\cancel{(x+7)}(x+7)} \cdot \frac{3\cancel{(x+8)}}{\cancel{(x-8)}\cancel{(x-8)}} \\ &= \frac{3(x+2)}{x+7}, x \neq -8, -7, -2, 8 \end{aligned}$$

$$\text{c) } \frac{x+4y}{x-5y} \cdot \frac{x^2-25y^2}{x^2-16y^2} \div \frac{x+5y}{x-4y}$$

$$= \frac{x+4y}{x-5y} \cdot \frac{(x-5y)(x+5y)}{(x-4y)(x+4y)} \div \frac{x+5y}{x-4y}$$

Non-permissible values: $x = -5y, -4y, 4y, 5y$

$$\begin{aligned} &= \frac{\cancel{x+4y}}{\cancel{x-5y}} \cdot \frac{\cancel{(x-5y)}\cancel{(x+5y)}}{\cancel{(x-4y)}\cancel{(x+4y)}} \cdot \frac{\cancel{x+5y}}{\cancel{x-4y}} \\ &= 1, x \neq -5y, -4y, 4y, 5y \end{aligned}$$

$$\begin{aligned} \text{d)} \quad & \frac{(5c + 6d)^2}{3c - 4d} \cdot \frac{9c^2 - 16d^2}{25c^2 - 36d^2} \div \frac{3c + 4d}{5c - 6d} \\ &= \frac{(5c + 6d)^2}{3c - 4d} \cdot \frac{(3c - 4d)(3c + 4d)}{(5c - 6d)(5c + 6d)} \div \frac{3c + 4d}{5c - 6d} \\ & \text{Non-permissible values: } c = -\frac{4d}{3}, -\frac{6d}{5}, \frac{6d}{5}, \frac{4d}{3} \\ &= \frac{(5c + 6d)^2}{\cancel{3c - 4d}} \cdot \frac{\cancel{(3c - 4d)} \cdot \cancel{(3c + 4d)}}{(5c - 6d)(5c + 6d)} \cdot \frac{\cancel{5c - 6d}}{\cancel{3c + 4d}} \\ &= 5c + 6d, c \neq -\frac{4d}{3}, -\frac{6d}{5}, \frac{6d}{5}, \frac{4d}{3} \end{aligned}$$

13. Simplify each expression.

$$\begin{aligned} & \frac{x^2 - 2x - 8}{x + 5} \div \frac{x^2 - x - 6}{x^2 + 2x - 15} \\ & \frac{x^2 - x - 6}{x^2 + 2x - 15} \div \frac{x^2 - 2x - 8}{x + 5} \end{aligned}$$

Compare the solutions.

What do you notice?

$$\begin{aligned} & \frac{x^2 - 2x - 8}{x + 5} \div \frac{x^2 - x - 6}{x^2 + 2x - 15} \\ &= \frac{(x - 4)(x + 2)}{x + 5} \div \frac{(x - 3)(x + 2)}{(x + 5)(x - 3)} \end{aligned}$$

Non-permissible values:

$$x = -5, -2, 3$$

$$\begin{aligned} &= \frac{(x - 4)\cancel{(x + 2)}}{x + 5} \cdot \frac{\cancel{(x + 5)} \cdot \cancel{(x - 3)}}{\cancel{(x - 3)} \cdot \cancel{(x + 2)}} \\ &= x - 4, x \neq -5, -2, 3 \end{aligned}$$

The solutions are reciprocals.

$$\begin{aligned} & \frac{x^2 - x - 6}{x^2 + 2x - 15} \div \frac{x^2 - 2x - 8}{x + 5} \\ &= \frac{(x - 3)(x + 2)}{(x + 5)(x - 3)} \div \frac{(x - 4)(x + 2)}{x + 5} \end{aligned}$$

Non-permissible values:

$$x = -5, -2, 3, 4$$

$$\begin{aligned} &= \frac{\cancel{(x - 3)} \cdot \cancel{(x + 2)}}{\cancel{(x + 5)} \cdot \cancel{(x - 3)}} \cdot \frac{x + 5}{(x - 4)\cancel{(x + 2)}} \\ &= \frac{1}{x - 4}, x \neq -5, -2, 3, 4 \end{aligned}$$

C

14. a) Write a polynomial A so that the expression below simplifies to -1 .

$$\begin{aligned} & \frac{3n^2 + 2n - 8}{n^2 + 4n + 4} \cdot \frac{A}{3n^2 - n - 4}, n \neq -2, -1, \frac{4}{3} \\ &= \frac{\cancel{(3n - 4)} \cdot \cancel{(n + 2)}}{\cancel{(n + 2)}(n + 2)} \cdot \frac{A}{\cancel{(3n - 4)}(n + 1)} \\ &= \frac{A}{(n + 2)(n + 1)} \end{aligned}$$

Set the expression equal to -1 .

$$\begin{aligned} -1 &= \frac{A}{(n + 2)(n + 1)} \\ A &= -(n + 2)(n + 1) \\ A &= -(n^2 + 3n + 2) \\ A &= -n^2 - 3n - 2 \end{aligned}$$

b) Write a polynomial B so that the expression below simplifies to $\frac{1}{5}$.

$$\frac{x^2 + 6x + 9}{5x^2 + 15x} \div \frac{3x^2 + 11x + 6}{B}, x \neq -3, 0$$

$$= \frac{(x+3)(x+3)}{5x(x+3)} \div \frac{(x+3)(3x+2)}{B}$$

$$= \frac{\cancel{(x+3)} \cdot \cancel{(x+3)}}{5x \cdot \cancel{(x+3)}} \cdot \frac{B}{\cancel{(x+3)}(3x+2)}$$

$$= \frac{B}{5x(3x+2)}$$

Set the expression equal to $\frac{1}{5}$.

$$\frac{1}{5} = \frac{B}{5x(3x+2)}$$

$$B = \frac{5x(3x+2)}{5}$$

$$B = x(3x+2)$$

$$B = 3x^2 + 2x$$

Since B is in the denominator, $x \neq -\frac{2}{3}, 0$

15. Simplify: $\frac{1 - \frac{2}{a}}{1 - \frac{4}{a^2}}$

$$= \frac{a-2}{a}$$

$$= \frac{a^2-4}{a^2}$$

$$= \frac{a-2}{a} \cdot \frac{a^2}{a^2-4}$$

$$= \frac{\cancel{a-2}}{a} \cdot \frac{a^2}{\cancel{(a-2)}(a+2)}$$

Non-permissible values: $a = -2, 0, 2$

$$= \frac{a}{a+2}, a \neq -2, 0, 2$$