Lesson 6: Combinations

GOALS

- Explore strategies for determining the number of combinations of n elements taken r at a time.
- Determine the number of combinations of n elements taken r at a time. Solve problems involving combinations.

While a permutation is a selection and *ordering* of a group of objects, a combination is a selection of a group of objects where the order of the objects is *not* important. In other words, a combination is a permutation without the ordering of the objects.

For example, when we are selecting people to sit on a committee, inviting people to a party, or when the winning numbers for a lottery are selected, the order of the selection doesn't matter because there are no specific positions involved.

We can calculate the number of combinations of a selection of objects by using reasoning or by using a function on our calculator.

Example 1

Determine the number of ways that a coach can select 3 athletes from a team of 6 to represent the school at a competition.

Method 1

We can use the fundamental counting principle to determine the number of permutations for this situation and then divide out the "ordering" of the number of objects.

$$Number\ of\ combinations = \frac{number\ of\ permutations}{number\ of\ ways\ to\ order\ the\ objects}$$

If 3 students must be selected from 6, the number of combinations possible is:

$$\#\ combinations = \frac{\#\ of\ permutations\ of\ 3\ objects\ from\ 6}{\#\ of\ permutations\ of\ 3\ objects}$$

$nCr = \frac{n!}{r!(n-r)!}$

$\frac{6!}{3!(6-3)!} = \frac{6!}{3!3!}$

$= \frac{6\times5\times4}{3!}$ which equals 20.

$\frac{6\times5\times4\times3!}{3!\,3!} = \frac{6\times5\times4}{3!} = 20$ ways

Method 2

We can use the ${}_nC_r$ feature on our calculator, where we are selecting r objects from a larger group of n objects.

In this case, because we are selecting 3 students from a larger group of 6, we would enter into our calculator: ${}_6C_3$, which also equals 20.

${}_6C_3 = 20$

$nCr = \frac{n!}{r!(n-r)!}$