

Lesson 4: Permutations Involving Restrictions

GOAL:

- To determine the number of possible permutations where there are various restrictions on specific positions.

Some permutation problems contain special considerations called *restrictions*.

A *restriction* can take on many forms, but at its simplest level it is a rule that forbids you from putting an item in a certain position in a permutation. Examine the next examples for more.

Example 1

A group of 18 people get together weekly to play board games. This group of people has a leader and a treasurer.

- a) In how many ways can the two roles be filled from the 18 people available?

$$\begin{array}{l} 18 \text{ leaders} \qquad 17 \text{ treasurers} \\ 18 \times 17 = 306 \end{array}$$

- b) In how many ways can the two roles be filled from the 18 people if one of the members refuses to be the leader?

$$\begin{array}{l} \text{leaders } 17 \\ 17 \times 17 = 289 \end{array}$$

Example 2

How many permutations of the word ORANGE are there that begin with a vowel?

$$\begin{array}{l} \text{6 letters} \\ \text{O A E} \quad 5 \quad 4 \quad 3 \quad 2 \quad 1 \quad 3 \times 5! \\ \underline{\quad 3} \\ 3 \times 5 \times 4 \times 3 \times 2 \times 1 = 360 \end{array}$$

Example 3

How many permutations for the word ORANGE are there that begin AND end with a vowel?

$$\begin{array}{l} \text{Begin} \qquad \qquad \qquad \text{end} \\ \cdot 3 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \\ \uparrow \\ 3 \times 4! \times 2 \\ = 144 \end{array}$$