

Lesson 2.1 Exercises, pages 89–94

A

3. Write the absolute value of each number.

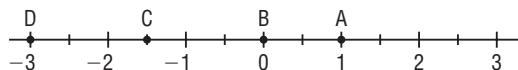
a) 15 $|15| = 15$

b) -7.6 $|-7.6| = 7.6$

c) 0 $|0| = 0$

d) $-\frac{7}{2}$ $|\frac{-7}{2}| = \frac{7}{2}$

4. Determine the absolute value of the number represented by each point on the number line.



A: $|1| = 1$ B: $|0| = 0$ C: $|-1.5| = 1.5$ D: $|-3| = 3$

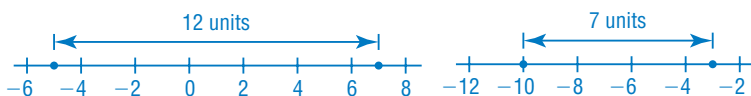
5. Use absolute value to determine the distance between each pair of numbers on a number line. Sketch a number line to illustrate the solution.

a) -5 and 7

b) -3 and -10

$$\begin{aligned} |7 - (-5)| &= |12| \\ &= 12 \end{aligned}$$

$$\begin{aligned} |-10 - (-3)| &= |-7| \\ &= 7 \end{aligned}$$



6. Evaluate.

a) $\sqrt{5^2}$

$$\begin{aligned}\text{Since } \sqrt{x^2} &= |x|, \\ \sqrt{5^2} &= |5| \\ &= 5\end{aligned}$$

b) $\sqrt{(-4)^2}$

$$\begin{aligned}\text{Since } \sqrt{x^2} &= |x|, \\ \sqrt{(-4)^2} &= |-4| \\ &= 4\end{aligned}$$

c) $\sqrt{10.1^2}$

$$\begin{aligned}\text{Since } \sqrt{x^2} &= |x|, \\ \sqrt{10.1^2} &= |10.1| \\ &= 10.1\end{aligned}$$

d) $\sqrt{(-11.4)^2}$

$$\begin{aligned}\text{Since } \sqrt{x^2} &= |x|, \\ \sqrt{(-11.4)^2} &= |-11.4| \\ &= 11.4\end{aligned}$$

B

7. a) Determine each absolute value.

$$|-11|, |12|, |0|, |4|, |-5|$$

$$\begin{aligned}|-11| &= 11 & |12| &= 12 & |0| &= 0 \\ |4| &= 4 & |-5| &= 5\end{aligned}$$

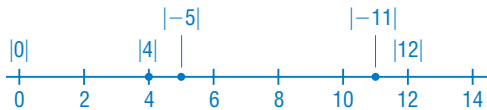
b) Order the numbers in part a from least to greatest.

Show the solution on a number line.

From least to greatest:

$$0 < 4 < 5 < 11 < 12$$

That is, $|0| < |4| < |-5| < |-11| < |12|$



8. For each pair of numbers, write two expressions to represent the distance between the numbers on a number line, then determine this distance.

a) $-2\frac{3}{4}$ and $5\frac{1}{4}$

$$\left| -2\frac{3}{4} - 5\frac{1}{4} \right| \text{ and } \left| 5\frac{1}{4} - \left(-2\frac{3}{4} \right) \right|, \text{ or } \left| 5\frac{1}{4} + 2\frac{3}{4} \right|$$

$$\begin{aligned}\left| -2\frac{3}{4} - 5\frac{1}{4} \right| &= \left| -\frac{11}{4} - \frac{21}{4} \right| \\ &= \left| -\frac{32}{4} \right| \\ &= |-8| \\ &= 8\end{aligned}$$

The numbers are 8 units apart on a number line.

b) $-5\frac{3}{5}$ and $-3\frac{7}{10}$

$$\left| -5\frac{3}{5} - \left(-3\frac{7}{10}\right) \right|, \text{ or } \left| -5\frac{3}{5} + 3\frac{7}{10} \right|,$$

$$\text{and } \left| -3\frac{7}{10} - \left(-5\frac{3}{5}\right) \right|, \text{ or } \left| -3\frac{7}{10} + 5\frac{3}{5} \right|$$

$$\begin{aligned} \left| -5\frac{3}{5} + 3\frac{7}{10} \right| &= \left| -\frac{28}{5} + \frac{37}{10} \right| \\ &= \left| -\frac{56}{10} + \frac{37}{10} \right| \\ &= \left| -\frac{19}{10} \right| \\ &= \frac{19}{10}, \text{ or } 1\frac{9}{10} \end{aligned}$$

The numbers are $1\frac{9}{10}$ units apart on a number line.

c) 12.47 and -8.23

$$\left| 12.47 - (-8.23) \right|, \text{ or } \left| 12.47 + 8.23 \right|, \text{ and } \left| -8.23 - 12.47 \right|$$

$$\begin{aligned} \left| 12.47 + 8.23 \right| &= \left| 20.7 \right| \\ &= 20.7 \end{aligned}$$

The numbers are 20.7 units apart on a number line.

9. Order the absolute values of the numbers in this set from greatest to least. Describe the strategy you used.

$$8.5, -8\frac{1}{3}, -10.2, -0.2, 0.1, 9.8$$

$$\left| 8.5 \right| = 8.5 \quad \left| -8\frac{1}{3} \right| = 8\frac{1}{3} \quad \left| -10.2 \right| = 10.2$$

$$\left| -0.2 \right| = 0.2 \quad \left| 0.1 \right| = 0.1 \quad \left| 9.8 \right| = 9.8$$

From greatest to least:

$$10.2 > 9.8 > 8.5 > 8\frac{1}{3} > 0.2 > 0.1$$

$$\text{That is, } \left| -10.2 \right|, \left| 9.8 \right|, \left| 8.5 \right|, \left| -8\frac{1}{3} \right|, \left| -0.2 \right|, \left| 0.1 \right|$$

I determined the absolute value of each number, then ordered the results from greatest to least.

10. The square of a number is 36. What is the absolute value of the number? Show your reasoning.

$$\begin{aligned} 36 &= 6^2 & 36 &= (-6)^2 \\ \sqrt{36} &= \sqrt{6^2} & \sqrt{36} &= \sqrt{(-6)^2} \\ &= \left| 6 \right| & &= \left| -6 \right| \\ &= 6 & &= 6 \end{aligned}$$

So, the absolute value of the number is 6.

11. Evaluate.

$$\begin{aligned} \text{a) } \sqrt{\left(3\frac{1}{8}\right)^2} \\ &= \left| 3\frac{1}{8} \right| \\ &= 3\frac{1}{8} \end{aligned}$$

$$\begin{aligned} \text{b) } \sqrt{\left(-2\frac{5}{6}\right)^2} \\ &= \left| -2\frac{5}{6} \right| \\ &= 2\frac{5}{6} \end{aligned}$$

$$\begin{aligned} \text{c) } & \sqrt{(9 - 11)^2} \\ &= \sqrt{(-2)^2} \\ &= |-2| \\ &= 2 \end{aligned}$$

$$\begin{aligned} \text{d) } & \sqrt{(13.5 - 16.1)^2} \\ &= \sqrt{(-2.6)^2} \\ &= |-2.6| \\ &= 2.6 \end{aligned}$$

12. Evaluate.

$$\begin{aligned} \text{a) } & |5(4) - 3| \\ &= |20 - 3| \\ &= |17| \\ &= 17 \end{aligned}$$

$$\begin{aligned} \text{b) } & 3|10 - 15| \\ &= 3|-5| \\ &= 3(5) \\ &= 15 \end{aligned}$$

$$\begin{aligned} \text{c) } & 10 - |5 - 8| \\ &= 10 - |-3| \\ &= 10 - 3 \\ &= 7 \end{aligned}$$

$$\begin{aligned} \text{d) } & |6 + (-10)| - |5 - 7| \\ &= |-4| - |-2| \\ &= 4 - 2 \\ &= 2 \end{aligned}$$

$$\begin{aligned} \text{e) } & 3|-2| - 4|3| \\ &= 3(2) - 4(3) \\ &= 6 - 12 \\ &= -6 \end{aligned}$$

$$\begin{aligned} \text{f) } & -3|4 - 1| + 2|1 - 4| \\ &= -3|3| + 2|-3| \\ &= -3(3) + 2(3) \\ &= -9 + 6 \\ &= -3 \end{aligned}$$

13. Evaluate.

$$\begin{aligned} \text{a) } & \sqrt{(5.1 - 2.3)^2} - |5.1 - 2.3| \\ &= \sqrt{2.8^2} - |2.8| \\ &= |2.8| - |2.8| \\ &= 0 \end{aligned}$$

$$\begin{aligned} \text{b) } & 5.1 - 2.3 - |5.1 - 2.3| \\ &= 2.8 - |2.8| \\ &= 2.8 - 2.8 \\ &= 0 \end{aligned}$$

$$\begin{aligned} \text{c) } & \frac{|2 - (-8)|}{|3| - |-2|} \\ &= \frac{|10|}{3 - 2} \\ &= \frac{10}{1} \\ &= 10 \end{aligned}$$

$$\begin{aligned} \text{d) } & |5 - 4|(5 + 4) - 2(5 + 4) \\ &= |1|(9) - 2(9) \\ &= 1(9) - 18 \\ &= 9 - 18 \\ &= -9 \end{aligned}$$

$$\begin{aligned}
 \text{e) } & \left| \frac{2}{3} \left| -\frac{5}{8} - \frac{1}{4} \right| \right| \\
 &= \left| \frac{2}{3} \left| -\frac{5}{8} - \frac{2}{8} \right| \right| \\
 &= \left| \frac{2}{3} \left| -\frac{7}{8} \right| \right| \\
 &= \frac{2}{3} \left(\frac{7}{8} \right) \\
 &= \frac{7}{12}
 \end{aligned}$$

$$\begin{aligned}
 \text{f) } & \left| 1\frac{3}{4} - 2\frac{1}{2} \right| - \left| \frac{3}{4} - \frac{1}{2} \right| \\
 &= \left| \frac{7}{4} - \frac{5}{2} \right| - \left| \frac{3}{4} - \frac{2}{4} \right| \\
 &= \left| \frac{7}{4} - \frac{10}{4} \right| - \left| \frac{1}{4} \right| \\
 &= \left| -\frac{3}{4} \right| - \frac{1}{4} \\
 &= \frac{3}{4} - \frac{1}{4} \\
 &= \frac{2}{4}, \text{ or } \frac{1}{2}
 \end{aligned}$$

14. Evaluate each expression for the given value of x .

$$\begin{aligned}
 \text{a) } & |x^2 + 6x - 5|, x = 2 \\
 &= |2^2 + 6(2) - 5| \\
 &= |4 + 12 - 5| \\
 &= |11| \\
 &= 11
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & |-3x^2 + 7x + 1|, x = 3 \\
 &= |-3(3)^2 + 7(3) + 1| \\
 &= |-27 + 21 + 1| \\
 &= |-5| \\
 &= 5
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } & |x^3 - 5x - 2|, x = -3 \\
 &= |(-3)^3 - 5(-3) - 2| \\
 &= |-27 + 15 - 2| \\
 &= |-14| \\
 &= 14
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } & |5x^3 + 2x + 7|, x = -1 \\
 &= |5(-1)^3 + 2(-1) + 7| \\
 &= |-5 - 2 + 7| \\
 &= |0| \\
 &= 0
 \end{aligned}$$

C

15. Two integers, a and b , are between -5 and 5 . The sum of the absolute values of the integers is 4 . What might the integers be? How many answers are possible?

Integers between -5 and 5 are: $-4, -3, -2, -1, 0, 1, 2, 3, 4$

$$|a| + |b| = 4$$

So, I test all pairs of these integers using the equation above.

Possible pairs of integers are: $a = 4, b = 0$; $a = 3, b = 1$; $a = 2, b = 2$; $a = 1, b = 3$; $a = 0, b = 4$; $a = -4, b = 0$; $a = -3, b = -1$; $a = -2, b = -2$; $a = -1, b = -3$; $a = 0, b = -4$; $a = -3, b = 1$; $a = -1, b = 3$; $a = 3, b = -1$; $a = 1, b = -3$; $a = -2, b = 2$; $a = 2, b = -2$

There are 16 possible answers.

- 16.** When 7 is added to an integer, x , the absolute value of the sum is 12. Determine a value for x . How many values of x are possible? Show what you did to solve the problem.

Write, then solve an equation: $|x + 7| = 12$

Since $|12| = 12$ and $|-12| = 12$,

then, $x + 7 = 12$ or $x + 7 = -12$
 $x = 5$ $x = -19$

So, two values of x are possible: 5 or -19