

## Lesson 7.5 Exercises, pages 583–590

### A

3. Identify the non-permissible values of the variable in each equation.

a)  $5 = \frac{3x}{x}$

$x = 0$

b)  $\frac{2}{x-1} = \frac{6}{x}$

$x = 0$  and  $x = 1$

c)  $\frac{3}{6-x} = \frac{2}{x+5}$

$x = -5$  and  $x = 6$

d)  $\frac{-1}{3x+6} = \frac{2x}{x-2}$

$x = -2$  and  $x = 2$

4. Solve each equation.

a)  $\frac{4}{5} = \frac{8}{d}$

Non-permissible value:  $d = 0$

Common denominator:  $5d$

$$\cancel{5}d\left(\frac{4}{\cancel{5}}\right) = 5\cancel{d}\left(\frac{8}{\cancel{d}}\right)$$

$$4d = 40$$

$$d = 10$$

b)  $\frac{e}{3} = \frac{12}{e}$

Non-permissible value:  $e = 0$

Common denominator:  $3e$

$$\cancel{3}e\left(\frac{e}{\cancel{3}}\right) = 3\cancel{e}\left(\frac{12}{\cancel{e}}\right)$$

$$e^2 = 36$$

$$e = 6 \text{ or } e = -6$$

Solutions are  $e = -6$  and  $e = 6$ .

5. Solve each equation.

a)  $\frac{3}{2z} = \frac{4}{3z} - \frac{1}{2}$

Non-permissible value:  $z = 0$

Common denominator:  $6z$

$$3 \cdot 6z \left( \frac{3}{2z} \right) = 2 \cdot 6z \left( \frac{4}{3z} \right) - 3 \cdot 6z \left( \frac{1}{2} \right)$$

$$9 = 8 - 3z$$

$$3z = -1$$

$$z = -\frac{1}{3}$$

b)  $2 - \frac{4}{x} = \frac{6}{x^2}$

Non-permissible value:  $x = 0$

Common denominator:  $x^2$

$$x^2(2) - x^2 \left( \frac{4}{x} \right) = x^2 \left( \frac{6}{x^2} \right)$$

$$2x^2 - 4x = 6$$

$$2x^2 - 4x - 6 = 0$$

$$2(x^2 - 2x - 3) = 0$$

$$2(x - 3)(x + 1) = 0$$

$$x = 3 \text{ or } x = -1$$

Solutions are  $x = -1$  and  $x = 3$ .

## B

6. Solve each equation.

a)  $\frac{3}{q-2} = \frac{5}{q+4}$

Non-permissible values:  $q = 2$  and  $q = -4$

Common denominator:  $(q-2)(q+4)$

$$(\cancel{q-2})(q+4) \left( \frac{3}{\cancel{q-2}} \right) = (q-2)(\cancel{q+4}) \left( \frac{5}{\cancel{q+4}} \right)$$

$$3q + 12 = 5q - 10$$

$$22 = 2q$$

$$q = 11$$

b)  $\frac{3}{2s-4} = \frac{3}{s-2}$

Non-permissible value:  $s = 2$

The numerators are the same, so equate the denominators.

$$2s - 4 = s - 2$$

$$s = 2$$

$s = 2$  is a non-permissible value, so the equation has no solution.

7. Solve each equation.

a)  $\frac{a-1}{a-3} = \frac{a+1}{a-4}$

Non-permissible values:  $a = 3$  and  $a = 4$

Common denominator:  $(a-3)(a-4)$

$$(\cancel{a-3})(a-4) \left( \frac{a-1}{\cancel{a-3}} \right) = (a-3)(\cancel{a-4}) \left( \frac{a+1}{\cancel{a-4}} \right)$$

$$(a-4)(a-1) = (a-3)(a+1)$$

$$a^2 - 5a + 4 = a^2 - 2a - 3$$

$$-3a = -7$$

$$a = \frac{7}{3}$$

$$\text{b) } \frac{2w + 1}{w - 4} = \frac{4w - 3}{2w + 1}$$

Non-permissible values:  $w = 4$  and  $w = -\frac{1}{2}$

Common denominator:  $(w - 4)(2w + 1)$

$$\cancel{(w - 4)}(2w + 1)\left(\frac{2w + 1}{\cancel{w - 4}}\right) = (w - 4)\cancel{(2w + 1)}\left(\frac{4w - 3}{\cancel{2w + 1}}\right)$$

$$(2w + 1)(2w + 1) = (w - 4)(4w - 3)$$

$$4w^2 + 4w + 1 = 4w^2 - 19w + 12$$

$$23w = 11$$

$$w = \frac{11}{23}$$

$$\text{c) } \frac{6}{2x^2 + 2x} = \frac{x - 2}{x + 1}$$

$$\frac{6}{2x(x + 1)} = \frac{x - 2}{x + 1}$$

Non-permissible values:  $x = -1$  and  $x = 0$

Common denominator:  $2x(x + 1)$

$$\cancel{2x(x + 1)}\left(\frac{6}{\cancel{2x(x + 1)}}\right) = 2x\cancel{(x + 1)}\left(\frac{x - 2}{\cancel{x + 1}}\right)$$

$$6 = 2x^2 - 4x$$

$$2x^2 - 4x - 6 = 0$$

$$2(x^2 - 2x - 3) = 0$$

$$2(x - 3)(x + 1) = 0$$

$$x = 3 \text{ or } x = -1$$

$x = -1$  is a non-permissible value.

So, the only solution is  $x = 3$ .

8. Here is a student's solution for solving a rational equation.

Identify the error in the solution. Write a correct solution.

$$\frac{1}{8} = 1 + \frac{2}{x}$$

$$\frac{1}{8} = \frac{3}{x}$$

$$\cancel{8}x\left(\frac{1}{\cancel{8}}\right) = \cancel{8}x\left(\frac{3}{\cancel{x}}\right)$$

$$x = 24$$

There is an error in line 2: the student added  $1 + \frac{2}{x}$  without using a common denominator. Correct solution:

Non-permissible value:  $x = 0$

Common denominator:  $8x$

$$\frac{1}{8} = 1 + \frac{2}{x}$$

$$\frac{1}{8} = \frac{x + 2}{x}$$

$$\cancel{8}x\left(\frac{1}{\cancel{8}}\right) = \cancel{8}x\left(\frac{x + 2}{\cancel{x}}\right)$$

$$x = 8x + 16$$

$$7x = -16$$

$$x = -\frac{16}{7}$$

9. Solve each equation.

a)  $\frac{1}{x} + \frac{1}{x-3} = \frac{x-2}{x-3}$

Non-permissible values:  $x = 0$  and  $x = 3$

$$\frac{1}{x} = \frac{x-2}{x-3} - \frac{1}{x-3}$$

$$\frac{1}{x} = \frac{x-3}{x-3}$$

$$\frac{1}{x} = 1$$

$$x = 1$$

b)  $\frac{1}{u-2} + \frac{u-1}{u^2-4} = \frac{u+4}{u+2}$

$$\frac{1}{u-2} + \frac{u-1}{(u-2)(u+2)} = \frac{u+4}{u+2}$$

Non-permissible values:  $u = -2$  and  $u = 2$

Common denominator:  $(u-2)(u+2)$

$$\begin{aligned} \cancel{(u-2)}(u+2)\left(\frac{1}{\cancel{u-2}}\right) + \cancel{(u-2)}(u+2)\left(\frac{u-1}{\cancel{(u-2)}(u+2)}\right) \\ = (u-2)\cancel{(u+2)}\left(\frac{u+4}{\cancel{u+2}}\right) \end{aligned}$$

$$u+2+u-1 = (u-2)(u+4)$$

$$2u+1 = u^2+2u-8$$

$$u^2-9 = 0$$

$$u = 3 \text{ or } u = -3$$

Solutions are  $u = -3$  and  $u = 3$ .

c)  $\frac{6b^2}{b^2-25} + \frac{4b}{5-b} = \frac{b}{b+5}$

$$\frac{6b^2}{(b-5)(b+5)} - \frac{4b}{b-5} = \frac{b}{b+5}$$

Non-permissible values:  $b = -5$  and  $b = 5$

Common denominator:  $(b-5)(b+5)$

$$\begin{aligned} \cancel{(b-5)}(b+5)\left(\frac{6b^2}{\cancel{(b-5)}(b+5)}\right) - \cancel{(b-5)}(b+5)\left(\frac{4b}{\cancel{b-5}}\right) \\ = (b-5)\cancel{(b+5)}\left(\frac{b}{\cancel{b+5}}\right) \end{aligned}$$

$$6b^2 - 4b(b+5) = b^2 - 5b$$

$$6b^2 - 4b^2 - 20b = b^2 - 5b$$

$$b^2 - 15b = 0$$

$$b(b-15) = 0$$

$$b = 0 \text{ or } b = 15$$

Solutions are  $b = 0$  and  $b = 15$ .

$$d) \frac{3z-1}{2z+1} + \frac{1}{6} = \frac{2z-1}{2z+1} + \frac{z+1}{z+3}$$

$$\frac{3z-1}{2z+1} - \frac{2z-1}{2z+1} + \frac{1}{6} = \frac{z+1}{z+3}$$

$$\frac{z}{2z+1} + \frac{1}{6} = \frac{z+1}{z+3}$$

Non-permissible values:  $z = -\frac{1}{2}$  and  $z = -3$

Common denominator:  $6(2z+1)(z+3)$

$$6(2z+1)(z+3)\left(\frac{z}{2z+1}\right) + 6(2z+1)(z+3)\left(\frac{1}{6}\right) = 6(2z+1)(z+3)\left(\frac{z+1}{z+3}\right)$$

$$6z(z+3) + (2z+1)(z+3) = (12z+6)(z+1)$$

$$6z^2 + 18z + 2z^2 + 7z + 3 = 12z^2 + 18z + 6$$

$$4z^2 - 7z + 3 = 0$$

$$(4z-3)(z-1) = 0$$

$$z = \frac{3}{4} \text{ or } z = 1$$

Solutions are  $z = \frac{3}{4}$  and  $z = 1$ .

10. Solve each equation.

$$a) \frac{b}{b^2-4} = \frac{2}{b^2-b-6}$$

$$\frac{b}{(b-2)(b+2)} = \frac{2}{(b-3)(b+2)}$$

Non-permissible values:  $b = 2$ ,  $b = -2$ , and  $b = 3$

Common denominator:  $(b-2)(b+2)(b-3)$

$$(b-2)(b+2)(b-3)\left(\frac{b}{(b-2)(b+2)}\right) = (b-2)(b+2)(b-3)\left(\frac{2}{(b-3)(b+2)}\right)$$

$$b^2 - 3b = 2b - 4$$

$$b^2 - 5b + 4 = 0$$

$$(b-4)(b-1) = 0$$

$$b = 4 \text{ or } b = 1$$

Solutions are  $b = 4$  and  $b = 1$ .

$$b) \frac{16}{2g^2+2g-12} = \frac{6}{g^2-9}$$

$$\frac{16}{2(g^2+g-6)} = \frac{6}{(g-3)(g+3)}$$

$$\frac{16}{2(g+3)(g-2)} = \frac{6}{(g-3)(g+3)}$$

Non-permissible values:  $g = -3$ ,  $g = 2$ , and  $g = 3$

Common denominator:  $2(g+3)(g-3)(g-2)$

$$2(g+3)(g-3)(g-2)\left(\frac{16}{2(g+3)(g-2)}\right) = 2(g+3)(g-3)(g-2)\left(\frac{6}{(g-3)(g+3)}\right)$$

$$16g - 48 = 12g - 24$$

$$4g = 24$$

$$g = 6$$

$$c) \frac{n}{n+1} + \frac{3n+5}{n^2+4n+3} = \frac{2}{n+3}$$

$$\frac{n}{n+1} + \frac{3n+5}{(n+1)(n+3)} = \frac{2}{n+3}$$

Non-permissible values:  $n = -1$  and  $n = -3$

Common denominator:  $(n+1)(n+3)$

$$\begin{aligned} \cancel{(n+1)}(n+3)\left(\frac{n}{\cancel{n+1}}\right) + \cancel{(n+1)}\cancel{(n+3)}\left(\frac{3n+5}{\cancel{(n+1)}\cancel{(n+3)}}\right) \\ = (n+1)\cancel{(n+3)}\left(\frac{2}{\cancel{n+3}}\right) \end{aligned}$$

$$n^2 + 3n + 3n + 5 = 2n + 2$$

$$n^2 + 4n + 3 = 0$$

$$(n+1)(n+3) = 0$$

$$n = -1 \text{ or } n = -3$$

$n = -1$  and  $n = -3$  are non-permissible values.

So, the equation has no solution.

11. The solutions of the equation  $4 = x + \frac{8}{x+m}$  are  $x = 3$  and  $x = -4$ . Determine the value of  $m$ . Show how you can verify your answer.

Common denominator:  $x + m$

$$(x+m)4 = (x+m)x + \cancel{(x+m)}\left(\frac{8}{\cancel{x+m}}\right)$$

$$4x + 4m = x^2 + mx + 8$$

$$x^2 + mx - 4x + 8 - 4m = 0$$

$$x^2 + x(m-4) + (8-4m) = 0$$

An equation with roots  $x = 3$  and  $x = -4$  is:

$$(x-3)(x+4) = 0$$

$$x^2 + x - 12 = 0$$

Compare the equations:  $x^2 + x(m-4) + (8-4m) = 0$

$$x^2 + x - 12 = 0$$

So,  $m-4 = 1$  and  $8-4m = -12$

$$m = 5$$

$$m = 5$$

The value of  $m$  is 5.

To verify the answer, substitute  $m = 5$  in the original equation, then solve the equation.

12. The measure,  $d$  degrees, of each angle in a regular polygon with  $n$  sides is given by the equation  $d = 180 - \frac{360}{n}$ .

- a) What is the measure of each angle in a regular polygon with 15 sides?

Substitute  $n = 15$ :

$$d = 180 - \frac{360}{15}$$

$$d = 180 - 24$$

$$d = 156$$

Each angle measures  $156^\circ$ .

- b) When each angle in a regular polygon is  $162^\circ$ , how many sides does the polygon have?

Substitute  $d = 162$ :

$$162 = 180 - \frac{360}{n}$$

$$18 = \frac{360}{n}$$

$$n(18) = n\left(\frac{360}{n}\right)$$

$$18n = 360$$

$$n = 20$$

The polygon has 20 sides.

13. Without solving the equation, how do you know that the equation

$$\frac{12}{4x - 4} = \frac{4}{x - 1}$$
 has no solution?

Simplify the expression on the left.

$$\frac{\cancel{4}^{3}12}{\cancel{4}(x - 1)} = \frac{4}{x - 1}$$

$$\frac{3}{x - 1} = \frac{4}{x - 1}$$

Since the expressions have the same denominator but different numerators, I know the equation has no solution.

## C

14. a) Write a rational equation that has 4 as a solution.

Work backward.

$$x = 4$$

$$x = 7 - 3$$

$$x + 3 = 7$$

$$\frac{x + 3}{x + 3} = \frac{7}{x + 3}$$

$$1 = \frac{7}{x + 3}, x \neq -3$$

Write 4 as a difference of 2 numbers.

Take 3 to the left side.

Divide each side by  $x + 3$ .

Simplify.

- b) Write a rational equation that has  $-3$  as a solution and has 3 as an extraneous root.

$$\frac{1}{x - 3}$$
 has  $x = 3$  as a non-permissible value.

Let both rational expressions in the equation have denominator  $x - 3$ .

$$\frac{\quad}{x - 3} = \frac{\quad}{x - 3}$$

Write an equation that has  $x = 3$  and  $x = -3$  as roots:

$$x^2 = 9$$

So, the rational equation becomes:

$$\frac{x^2}{x - 3} = \frac{9}{x - 3}$$

This equation has  $x = 3$  and  $x = -3$  as roots. Since  $x = 3$  is a non-permissible value,  $x = 3$  is an extraneous root. The only solution is  $x = -3$ .

15. Solve each equation.

a)  $\frac{x+1}{x-3} = \frac{2x}{x+2}$

Non-permissible values:  $x = 3$  and  $x = -2$

Common denominator:  $(x-3)(x+2)$

$$\cancel{(x-3)}(x+2)\left(\frac{x+1}{\cancel{x-3}}\right) = (x-3)\cancel{(x+2)}\left(\frac{2x}{\cancel{x+2}}\right)$$

$$x^2 + 3x + 2 = 2x^2 - 6x$$

$$x^2 - 9x - 2 = 0 \quad \text{Use the quadratic formula.}$$

$$x = \frac{9 \pm \sqrt{(-9)^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{9 \pm \sqrt{89}}{2}$$

Solutions are  $x = \frac{9 + \sqrt{89}}{2}$  and  $x = \frac{9 - \sqrt{89}}{2}$ .

b)  $\frac{w^2}{w+4} = \frac{5}{3}$

Non-permissible value:  $w = -4$

Common denominator:  $3(w+4)$

$$3\cancel{(w+4)}\left(\frac{w^2}{\cancel{w+4}}\right) = 3\cancel{(w+4)}\left(\frac{5}{\cancel{3}}\right)$$

$$3w^2 = 5w + 20$$

$$3w^2 - 5w - 20 = 0 \quad \text{Use the quadratic formula.}$$

$$w = \frac{5 \pm \sqrt{(-5)^2 - 4(3)(-20)}}{2(3)}$$

$$w = \frac{5 \pm \sqrt{265}}{6}$$

Solutions are  $w = \frac{5 + \sqrt{265}}{6}$  and  $w = \frac{5 - \sqrt{265}}{6}$ .

c)  $\frac{1}{v} + \frac{1}{v-4} = 2$

Non-permissible values:  $v = 0$  and  $v = 4$

Common denominator:  $v(v-4)$

$$\cancel{v}(v-4)\left(\frac{1}{\cancel{v}}\right) + v\cancel{(v-4)}\left(\frac{1}{\cancel{v-4}}\right) = v(v-4)(2)$$

$$v - 4 + v = 2v^2 - 8v$$

$$2v^2 - 10v + 4 = 0$$

$$v^2 - 5v + 2 = 0 \quad \text{Use the quadratic formula.}$$

$$v = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(2)}}{2(1)}$$

$$v = \frac{5 \pm \sqrt{17}}{2}$$

Solutions are  $v = \frac{5 + \sqrt{17}}{2}$  and  $v = \frac{5 - \sqrt{17}}{2}$ .