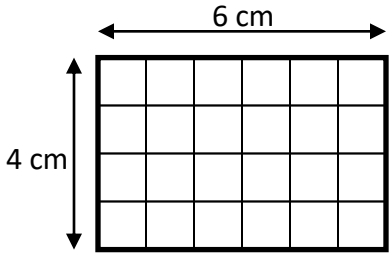
Applied Math 40S
Review: Area, Surface Area and Volume Primer

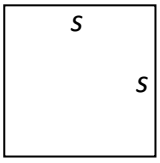
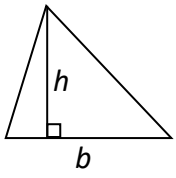
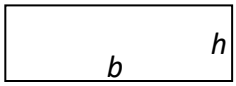
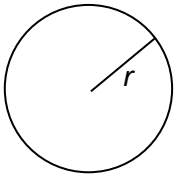
Area

Area is the measurement of the amount of 2-dimensional space that a 'flat' object occupies. It is measured in units squared (units^2). For example, the rectangle below has been filled with blocks that are one cm on each side. Each little block is referred to as a 'square centimeter'. (A square with centimeter-long sides).



Since there are 24 of these boxes contained in the rectangle, we say that the rectangle has an *area* of 24 *square centimeters*.

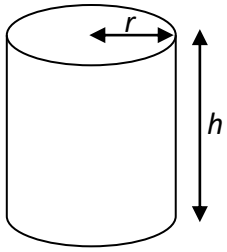
The short-cut to arrive at this number is to use the formula for the area of a rectangle, namely: $A_{rect} = LW$. By multiplying the length by the width, you can quickly and accurately calculate the area of a rectangle. Some important area formulas are:

Figure Name	Diagram	Formula	Figure Name	Diagram	Formula
square		$\boldsymbol{A = s^2}$	triangle		$\boldsymbol{A = \frac{1}{2}bh}$
rectangle		$\boldsymbol{A = bh}$	circle		$\boldsymbol{A = \pi r^2}$

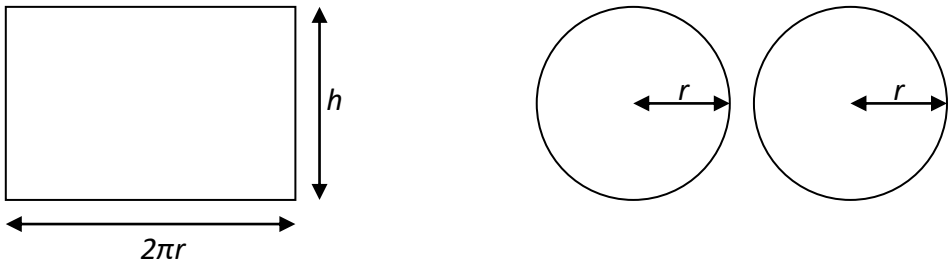
Area is commonly used to calculate things like amount of paint needed to cover a wall, or how much carpet is needed to cover a floor.

Surface Area

Surface area refers to the same concept as area, but it is the measurement of area covering a 3-dimensional shape. For example, take a look at the cylinder below:



If we break the cylinder apart we can see that it is made up of three 2-dimensional shapes: two circles and a rectangular 'centre part' that gets wrapped around to make the cylinder itself. Each of these shapes has an area. If you add up these areas you get the *surface area* of the whole figure.

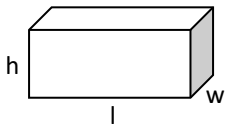
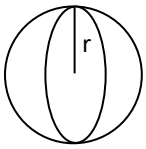
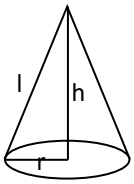
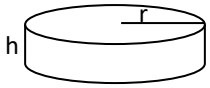


You can see that the formula for the surface area of a cylinder come from combining the area of all of its constituent parts. Surface area is used for the same type of applications as area (such as an amount of paint) and it typically applies to the 'outside' of a three-dimensional object. Surface area (like area) is measured in *units squared* (written $unit^2$).

If you were trying to find the surface area of an open-ended cylinder (like a soup can with no top) you would just omit that part of the formula (one of the πr^2's) making the formula:

$$SA_{cyl} = 2\pi rh + \pi r^2$$

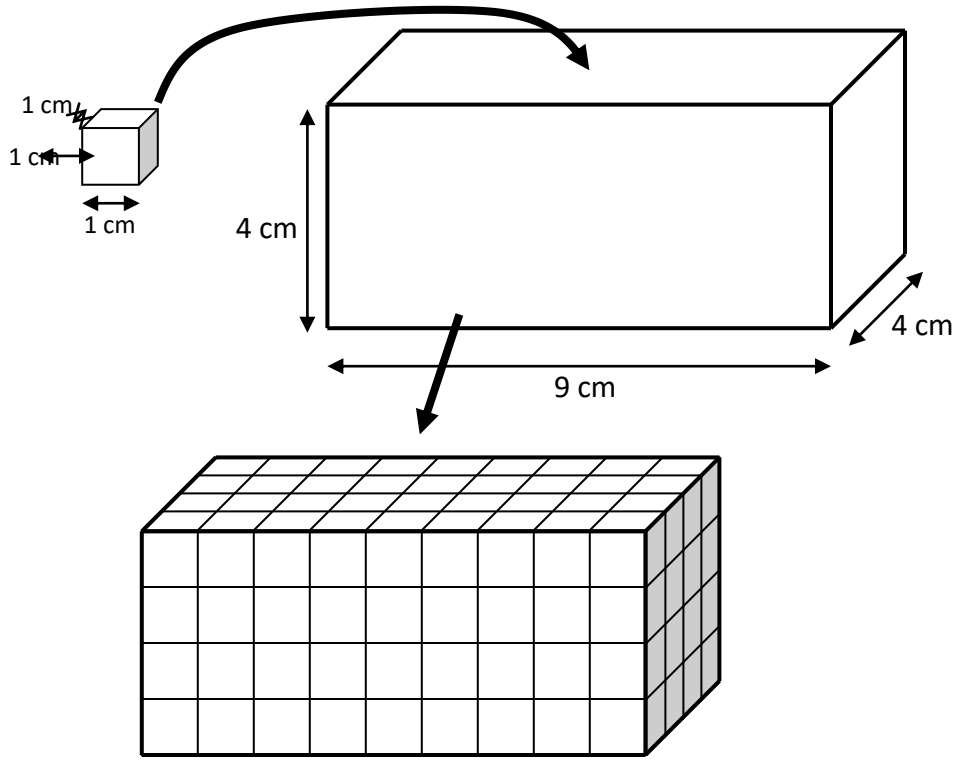
The formulas for the surface areas of some common objects are on the back of this page.

Figure Name	Diagram	Formula
Rectangular Solid (Box)		Top or Bottom = lw Front or Back = lh Either End = wh **SA = 2lw + 2lh + 2wh**
Sphere (Ball)		$\boldsymbol{SA = 4\pi r^2}$
Cone (yum)		Side = πrl Bottom = πr^2 **SA = $\boldsymbol{\pi rl + \pi r^2}$**
Cylinder (Can? Tube?)		Side = $2\pi rh$ Bottom or Top = πr^2 **SA = $\boldsymbol{2\pi rh + 2\pi r^2}$**

Volume

Volume is also used to measure 3-dimensional objects, but it measures the amount of space *inside* a 3-dimensional object (like how much air or water can go into an object).

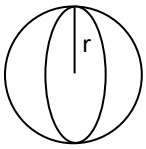
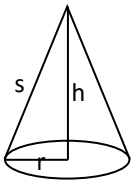
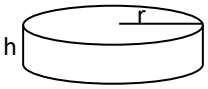
For a quick example, let's look at a box. To measure the volume, you fit as many *cubes* of a certain size inside of the box. For this example, we'll use a cube with sides of 1 cm. These are *cubic centimeters*. Volume is always measured in cubic units (units^3).



If you took the time to count all of the little cubes that fit in the box, you would see that there are 144 of them. This box has a *volume* of 144 *cubic centimeters*. This could also be written as 144 cm^3.

The faster way to calculate the volume of a *rectangular solid* (rectangular solid is a fancy term for a box) is to use the formula: $V_{box} = L \cdot W \cdot H$. So, in this case, we would get:
V = (9)(4)(4) or V = 144. This is much faster and more accurate than counting cubes!

Formulas to find the volume of common 3-dimensional objects are given on the back side of this page.

Figure Name	Diagram	Formula
Rectangular Solid (Box)		$V = lwh$
Sphere (Ball)		$V = \frac{4}{3}\pi r^3$
Cone (yum)		$V = \frac{1}{3}\pi r^2 h$
Cylinder (Can? Tube?)		$V = \pi r^2 h$

Buying in Whole Units, and Cost

Many design and measurement questions try to determine the amount of construction material to be ordered, and its cost.

It is important to note that you CANNOT purchase part units of supplies. For example, if you need 2.3 cans of paint, you need to buy 3 cans. If you need 64.2 floor tiles, you have to buy at least 63 (and maybe more, if they are sold by the case)!

When calculating the cost, it is common that you must add taxes as well. Remember that taxes in Manitoba are currently 8% for the Provincial Sales Tax (PST) and 5% to the Federal government for the Goods and Services Tax (GST).

Below are two questions off the January 2013 Applied Math Exam, with solutions shown. If you want to challenge yourself, cover up the solutions and try the questions yourself first!

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1. The Bertrands want to empty their circular swimming pool. There is 3 feet of water left in the pool which has a diameter of 16 feet. Using a pump which can remove 400 ft^3 of water per hour, how many hours will it take to remove all the water?

   Solution

   The pool is 'circular', but it also has a depth – that makes it a cylinder. First, find the volume of water remaining in the pool:

   $$V = \pi r^2 h$$
   $$V = \pi(8)^2(3)$$

   (The 8 is the **r**adius (half the diameter) and the 3 is the **h**eight of water remaining in the pool.)

   $$V = 603.2\ ft^3$$

   Now since the pump can remove 400 ft^3 per hour, divide the amount of water by the amount that can be removed in an hour:

   $$603.2 \div 400 \approx \mathbf{1.5}\ \boldsymbol{hours}$$

2. You have been asked to install floor tiles and paint your aunt's bathroom based on the following information:

- The floor measures 5 ft. × 7 ft.
- The walls are 8 ft. high.
- The door measures 80 in. × 30 in.
- The window measures 24 in. × 30 in.

a) You must cover the entire bathroom floor with tiles. Each tile measures 1 ft. × 1 ft. You will need an extra 5% of tiles to account for waste. How many tiles will you need to purchase for the project?

b) You must apply two coats of paint to the walls of the bathroom. The door and the window will not be painted. Determine the total area to be painted. How many cans of paint will you need to purchase if one can covers 100 ft^2? Show your work.

Solutions

a) The bathroom floor has an area of:

$$5(7) = 35\ ft^2$$

Since one tile covers one square foot, you are going to need 35 tiles, plus 5% extra tiles:

$$35 \times 1.05 = 36.75\ tiles$$

You can only buy whole tiles, so you will need **37 tiles**.

b) You have four walls. Two of the walls are 5 ft x 8 ft, and two of the walls are 7 ft by 8 ft. The area of all of the walls is:

$$2(5)(8) + 2(7)(8) = 192\ ft^2$$

The door has a measurement of 80 in by 30 in, convert these measurements to feet first, and then find the area of the door:

$$80 \div 12 = 6.67\ ft \qquad 30 \div 12 = 2.5\ ft$$

$$A = 6.67(2.5) = 16.68\ ft^2$$

Do the same for the window:

$$24 \div 12 = 2\ ft \qquad 30 \div 12 = 2.5\ ft$$

$$A = 2(2.5) = 5\ ft^2$$

Now subtract the area of the door and window from the area of the walls:

$$192 - 16.68 - 5 = 170.32\ ft^2\ (to\ be\ painted)$$

Since we need *2 coats*, multiply this area to be painted by 2:

$$170.32 \times 2 = 340.64\ ft^2$$

Finally, since a can covers 100 ft^2, we can calculate that we need:

$$340.64 \div 100 = 3.4064\ cans$$

And since we can't buy part of a can, our final answer is:

$$\boldsymbol{4\ cans}$$