If p then q. — conditional statement
If q then p — converse statement (switched)
If NOT p then NOT q → inverse statement
If NOT q then NOT p → contrapositive

Lesson Five: Logic Statements

Biconditional → p → q is true, q → p is True

GOAL: Identify and interpret different kinds of logic statements.

Conditional Statements

The following is a conditional statement:

"If today is Monday, then tomorrow is Tuesday". conclusion

A conditional statement is a sentence that has the form "If p ... then q".

The notation that denotes "If p, then q" is $p \rightarrow q$.

hypothesis

Ordinary statements can often be expressed in conditional form by carefully inserting the words "if" and "then". For example, if we start with "A square has four sides" we can write the conditional statement as "If a shape is a square, then it has four sides."

Hypothesis and Conclusion

hypothesis → Assumption

Conclusion → The result of a hypothesis

"If **today is Monday**, then **tomorrow is Tuesday**."

Hypothesis p **Conclusion** q

The phrase that follows the "If..." in a conditional statement is called the *hypothesis*.
The phrase that follows the "then..." in a conditional statement is called the *conclusion*.

Example 1

a) Write a conditional statement for "It's October so next month is November."

If $\underbrace{\text{it's October}}_{p}$, then $\underbrace{\text{next month is November}}_{q}$

b) Identify the hypothesis. (p) → "it's October

c) Identify the conclusion. (q) next month is November

NOTE: When the hypothesis (p) and the conclusion are True then the statement is True.

To disproves a statement by giving an example that shows it is not the case

To show a statement is NOT true (false) give a counterexample.

Counterexamples

An example that shows that statement is false is called a *counterexample*. The statement "If it is winter in Winnipeg, then it is January" is disproved by the counterexample that if it is winter in Winnipeg, it could be February.

"If it is winter in Winnipeg, it could be February." counterexample

Only ONE counter-example is needed to disprove a statement.

Example 2

Is the statement "If you are eating a doughnut, then you are at Tim Horton's" true? If it is false, provide a counterexample

If you are eating a doughnut, then you are at Duncan Donuts."

Converse - If q then p

The converse of a conditional statement is formed when the hypothesis and conclusion switch places.

Statement: If today is Monday, then tomorrow is Tuesday. True
Converse: If tomorrow is Tuesday, then today is Monday." True

Statement: If it is January, then it is winter in Winnipeg True
Converse: "If it is winter in Winnipeg, then it is January." False because winter could be in November or December, or January or February

counter example If its winter, then it could be February.

Note: The converse of a true statement may be true or false.
Which of the four statements above are true and which ones are false? For the statements that are false, provide a counter-example. If its winter, then it is February

nullify, negative, show the opposite

Inverse If NOT p then NOT q.

The inverse of a conditional statement is formed by *negating* both the hypothesis and conclusion of the original statement.

Statement: If today is Monday, then tomorrow is Tuesday. True
Inverse: If today is not Monday, then tomorrow is not Tuesday. True
negate the hypothesis negate the conclusion
Statement: If it is January, then it is winter in Winnipeg. True
Inverse: If it is not January, then it is not winter in Winnipeg. False

counterexample If it is not February, then it is not winter in Winnipeg

Note: The inverse of a true statement may be true or false. It's truth value will be the same as the *inverse*.
Which of the four statements above are true and which ones are false? For the statements that are false, provide a counter-example.

negating both - hypothesis and conclusion

Example 3

a) Write the inverse of this statement:
Statement: If I am at the University of Winnipeg, then I am downtown.

Inverse: If I am NOT at the U of W, then I am NOT downtown

b) Is the inverse true or false? How do you know? False, because the conclusion is NOT True.
→ You could be at the Bay and still be downtown.

Contrapositive → If NOT q then NOT p

The contrapositive of a conditional statement is formed by *negating* both the hypothesis and conclusion of the *converse* of that conditional statement. For example,

True Statement: If it is January, then it is winter in Winnipeg.
True Contrapositive: "If it is not winter in Winnipeg, then it is not January

Example 4

a) Write the contrapositive of this statement.
False Statement: "If I am at the University of Winnipeg, then I am downtown."

True Contrapositive: If I am NOT downtown, then I am NOT at U.ofW.

Note: The contrapositive of a true statement may be true or false. Its truth value will be the same as the *original conditional statement*.
Which of the four statements above are true and which ones are false? For the statements that are false, provide a counter-example.

If the original conditional statement is true, then its contrapositive will also be true.

If the converse of the conditional statement is true, then the inverse will also be true.

$p \Leftrightarrow q$ $\quad$ $\left.\begin{matrix} p \to q \to \text{True} \\ q \to p \to \text{True} \end{matrix}\right\}$ Biconditional

p if and only if q $\quad$ (p ~~iff~~ q)

Biconditional Statements

If a statement and its converse are ***both*** true, then a biconditional statement can be created. Biconditional statements use the phrase "***if and only if***".

Biconditional statements are often used to write definitions.

> Statement: If a shape has exactly three straight sides, then the shape is a triangle.
> Converse: "If a shape is a triangle, then the shape has exactly three straight sides."

Since the statement and its converse are both true we can write a biconditional statement using the phrase "***if and only if***":

A shape is a triangle if and only if it has exactly three straight sides.

A shape is a triangle iff it has three sides

Example 5

Verify that the following statement is true. Then write the converse and verify that the converse is true. If they are both true, re-write the statement as a biconditional statement.

converse

"If I earned the credit, then I passed the course."

Converse If I passed the course, then I will earn a credit.

Biconditional I will earn earn a credit if and only if I pass the course.

Example 6 Consider the original statement:

"If students are in grade 12, then they will graduate this June."

a) Write the converse of this statement.

If students graduate this year, then they are in Grade 12.

b) Determine if a biconditional statement is possible. If yes, write the biconditional statement. If not, provide a counterexample.

Biconditional students will graduate this year iff they are in grade 12.

counterexample;
students could be graduating from University.

Assignment 5

1. Consider the following statement:

   If you live in Canada, then you live in North America.

   a) State the hypothesis.
   b) State the conclusion.
   c) Is the statement true? If not, provide a counter-example.

2. Write each of the following statements in conditional ("*if ... then*") form.
   a) A half-empty glass is half-full.
   b) A pet that barks is a dog.
   c) A rhombus has equal opposite angles.

3. Ravi found that whenever he added two prime numbers, the sum was always even. State a counter-example to prove him wrong.

4. For each of the following statements, write the converse of the statement. Determine the truth of the statement and the truth of its converse. Provide a counter-example if necessary.
   a) If a shape is a square, then it has four sides.
   b) If a number is even, then it is divisible by 2.

5. For each of these statements, write the inverse. Determine the truth of the statement and the truth of its inverse. Provide a counter-example if necessary.
   a) If Melanie lives in Plum Coulee, then Melanie lives in Manitoba.
   b) If May 4 is Monday, then May 5 is Tuesday.
   c) If Fido has four legs, then Fido is a dog.
   d) If it is snow, then it can be melted.

6. For each of the following statements, write the contrapositive. Determine the truth of the statement and its contrapositive. Provide a counterexample if necessary.
   a) If you can operate a car, then you can fly a plane.
   b) If a child is less than 6 years old, then the child believes in the tooth fairy.
   c) If you are a successful basketball player in college, then you are taller than average.
   d) If you studied home economics in school, then you are a good cook.

7. Consider the original statement:

If a whole number is odd, then it is not evenly divisible by 2.

a) Write the converse of the statement.
b) Determine if a biconditional statement is possible. If yes, write the biconditional statement. If not, provide a counterexample.

8. Consider the original statement:

If a shape is a circle, then it has a single side that is a continuous curve.

a) Write the converse of the statement.
b) Determine if a biconditional statement is possible. If yes, write the biconditional statement. If not, provide a counterexample.

9. (Multiple Choice) Which of the following statements is ***not*** biconditional?
a) If a number is less than zero, then it is negative.
b) If a shape is a square, then it is a rectangle.
c) If $x + y = 3$, then $y = 3 - x$.
d) If a glass is half-empty, then it is half-full.

10. Your best friend, who always tells the truth, says she will meet you in the lab or in the library. You go to the library and she is not there. What can you conclude?