Lesson 3: Determining a Sinusoidal Equation

GOAL:

- Graph data and determine the sinusoidal function that best approximates the data.
- Determine a sinusoidal equation from a contextual problem.

As we've seen, the graphing calculator displays the equation of a sinusoidal function using the form: $y = a \sin(bx + c) + d$

In this section we will perform sinusoidal regressions to determine suitable sinusoidal equations that we will then use to solve problems.

Example 1

A weight is suspended by a spring. It is currently in its resting position (not moving) and is 0.50 metres above a table. The weight is pulled down 0.40 metres and is then released. The following information is obtained by observing its height above the table in relation to time, in seconds.

L1: x-axis

Time (s)	0	0.30	0.60	0.90	1.20
Height (m)	0.10	0.50	0.90	0.50	0.10

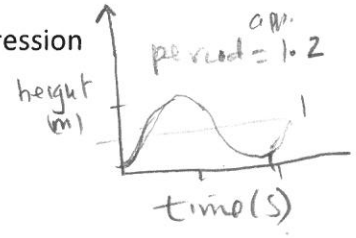
L2: y-axis

Window
xmin = −2
xmax = 4
ymin = −1
ymax = 3

KEYS: STAT | CALC → enter data. STAT | CALC | SinReg

a) Enter the data into the graphing calculator and determine a sinusoidal regression equation that best models the data.

$y = 0.4(\sin 5.24x - 1.57) + 0.50$



b) When will the weight be 0.75 m above the table for the first time?

(height)

$42 = 0.75$, CALC #5

$x = 0.43$ sec

c) How high will the weight be in 1 minute?

Use CALC 1: $x = 60$ seconds

$y = 0.10$ metres

d) What is the period of this function? What does the period represent in this context?

$b = 5.24$

$\text{period} = \frac{2\pi}{b}$

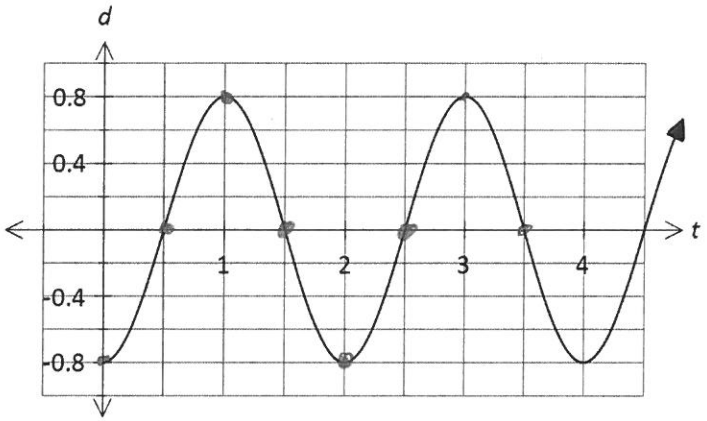
$= \frac{2\pi}{5.24}$

$= 1.20$ seconds

It takes 1.20 seconds for the weight to go up and down once.

Example 2

Alice is swinging on a playground swing. She starts by pushing the swing back 0.8 m from the swing's resting position. (Hint: Call this –0.8 m). The swing then takes one second to swing forward until it is 0.8 m in front of its resting position. She continues to swing at this rate for a long time, as shown on the graph below where d represents the distance from resting position and t represents time, in seconds.



NOTE: Select at least 5 points.

a) Place a dot on the graph at five of the 'main points' of the sinusoidal curve shown.

b) Fill in the table below with the coordinates of the five points that you placed on the graph.

L1	t	0	0.5	1	1.5	2
L2	d	−0.8	0	0.8	0	−0.8

Window
x-min = −2
x max = 4
y min = −2
y max = 2

c) Use your data to determine a sinusoidal regression equation that would represent this graph. Enter data into calculator STAT | EDIT, Then STAT | CALC | [C]

$y = 0.8 \sin(3.14x - 1.57) + 0$

d) When does the swing pass through 0.7 m for the first time?

let $y_2 = 0.7$

$x = 0.84$ seconds

e) State an appropriate domain and range for this situation.

For a 5 min swing time ⇒ 1 cycle = 2 seconds. period $\frac{2\pi}{\pi} = 2$

300 seconds = 150 swings

Domain = [0, 300]

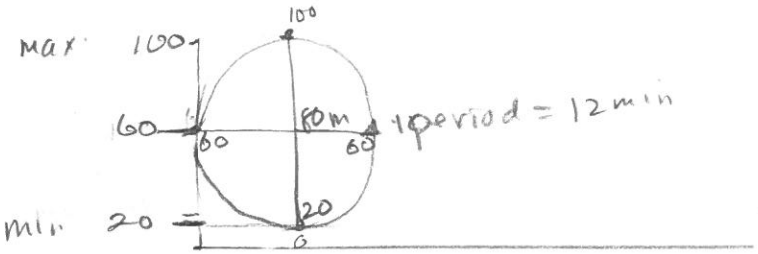
Range = [−0.8, 0.8]

Example 3:

The Ferris wheel was invented in 1893 by a man named George W. Ferris. The diameter of one of the first Ferris wheels was 80 metres and the maximum height that Ferris wheel achieved was 100 metres. The wheel rotated every 12 minutes.

a) Draw a sketch of this situation and determine a sinusoidal regression equation that best models this data. Show the data that you entered into your lists.

Time (mins)	0	3	6	9	12
Height (m)	20	60	100	60	20



window
x min = 0
x max = 30
y min = 0
y max = 120

EDIT | CALC | SinReg

$y = 40\sin(0.52x - 1.57) + 60$

(b)

b) Assuming that a person got on the ride at time 0, how high are they 5 minutes after the wheel starts rotating? $x = 5$ (use CALC #5)

$y = 94.64$ m

c) How many minutes after the wheel starts rotating does the cart first reach 25 metres from the ground? let $y_2 = 25$ use CALC #5

$x = 0.97$ minutes

d) State the period of this function and explain what it represents in this context.

$\text{Period} = \frac{2\pi}{b}$

$= \frac{2\pi}{0.52} \Rightarrow 12.08$ minutes

This rep. 1 complete rotation of the ferris wheel.

e) State a possible domain for this context.

Domain = [0, 30]