Lesson 2: Exploring Equations of Sinusoidal Functions

The graphing calculator uses the form $y = a \sin(bx + c) + d$ for the equation of a sinusoidal function. Where a, b, c, and d are variables which affect the size or position of the sinusoidal function.

GOALS:

- To determine the characteristics of sinusoidal functions by analyzing their equations.
- Match sinusoidal equations to their graphs.
- Solve problems given the equation of a sinusoidal function.

a: has the same value as the amplitude but while "a" may be positive or negative, the amplitude is always positive. Amplitude is a distance so it is always positive.

$$a = \frac{max - min}{2}$$

b: the *frequency* of the function, or how many cycles are repeated in a certain horizontal distance. As the period gets larger, the"b"value decreases. As the period gets smaller, the "b" value increases.

$$b = \frac{2\pi}{period} \quad \textbf{OR} \quad period = \frac{2\pi}{b}$$

c: is related to the horizontal shift. Your teacher can explain what a horizontal shift is. It will not be discussed in depth in this course.

d: is the value of the midline (median).

$$d = \frac{max + min}{2}$$

amplitude ↓ ↙ phase angle

$y = a\sin(b-c)+d$ ← d = midline

↑ horizontal shift ↑ vertical shift

period $= \frac{2\pi}{b}$

Example 1

Use a calculator or your knowledge of sinusoidal equations to complete the table below.

Equation	Minimum Value	Maximum Value	Amplitude	Equation of midline	Period
a) $y = -3\sin(3.14x - 2) + 1$ (a b c d)	−2	4	3	$y=1$	2
b) $y = 2\sin(0.25x + 4)$	−2	2	2	$y=4$	$\frac{2\pi}{0.25}$
c) $y = 4\sin x - 2$	−6	2	4	$y=-2$	2π
d) $y = -\sin(1.57x + 6) - 3$	−4	2	1	$y=-3$	4
e) $y = 0.5\sin(3x + 0) + 0$	−0.5	0.5	0.5	$y=0$	$\frac{2\pi}{3}$

$\frac{2\pi}{b} = \frac{2\pi}{\pi}$; $\frac{2\pi}{0.25}$; $\frac{2\pi}{1}$; $\frac{2\pi}{1.57}$

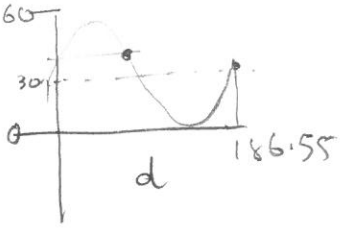
Example 2

A car is driving down the street and a pebble gets caught in one of its tire treads. The tire rotates and the height of the pebble varies sinusoidally with the horizontal distance. This situation is modelled by the equation:

$$h = 30\sin(0.0334d - 1.57) + 30$$

(a b c d; h; ↑ midline)

where d represents the distance the tire travels (in centimetres) and h represents the height of the pebble (in centimetres).



a) What is the maximum height that the pebble reaches?

60 cm

b) What is the period of this function and what does it represent in the context of the situation?

$\text{period} = \frac{2\pi}{b}$

$= \frac{2\pi}{0.0334} \Rightarrow 188.12$ cm

distance

c) How long (far has) is the pebble above a height of 40 cm?

$y \geq 40$

use CALC #5: 130.89 − 57.18 = 73.71 cm

d) Determine the circumference of the tire.

the circumference = 1 period

= 188.12 cm