Lesson 3: Using Regressions to Model Data with Polynomial Functions

GOALS:
- Use an appropriate regression to determine polynomial equations from a set of data.
- Use polynomial equations to solve problems.

When given a set of data, we can use our graphing calculator to first input the data and then inspect the resulting scatterplot (graph of the data points) to determine if they form a linear, a quadratic, or a cubic pattern. We can then use the graphing calculator to determine an equation that "fits" the data which will then let us solve more problems.

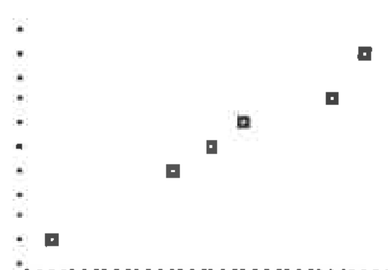
Example 1

Joseph wonders whether he can predict the size of a person's hand span based on the person's height. He records the measurements from 6 students in the table below:

Height (cm)	Hand Span (cm)
165	20
172	22
154	17
169	21
183	25
180	23

Enter the Height data in L1 and the Hand Span data in L2.

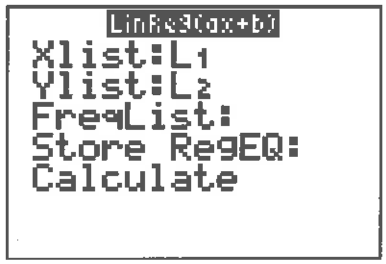
Plot the data: Turn STAT PLOT on. Make sure your windows are large enough to match the data values you entered in. You can also use ZOOM 9 and the calculator will set the window for you. Press GRAPH and look at the data. Does the data seem to follow a pattern? Which polynomial function pattern does the data seem to most closely resemble?



Now we will determine an equation that fits this data:

We will use a *Linear Regression* to find the line of best fit for this data. Press:

- STAT/CALC/4: LinReg/ENTER. You should see this on your screen:



Your teacher will guide you through the use of this menu.

After pressing 'Calculate', record the values you are given and also record them in the $y = ax + b$ form of the linear equation. This is the equation of the line that technology (your calculator) decided was the best fit for the data you entered in.

Equation:

Press GRAPH and you should see this line on the graph through the data values that were plotted. Notice how the line travels through most of the data points.

a) Suggest an appropriate domain and range for this function.

$y = 0.26x - 22.41$ → linear equation

Domain [154, 183] or $\{x / 154 \leq x \leq 183, x \in R\}$

Range [17, 25] or $\{y / 17 \leq y \leq 25, y \in R\}$

b) Predict the hand span for a person with a height of 167 cm. Use the 2nd, CALC, VALUE menu to enter $x = 167$

when $x = 167$, $y = 20.44$

c) Predict the height for a person with a hand span of 18.3 cm. Your teacher will guide you through this process.

when $x = 183.3$

height 158.68 a.

The same process can be used for data that displays either a quadratic or cubic pattern. The only difference is that after plotting and inspecting the data, we use a different regression option to create a graph that travels through most of the data points.

Example 2

Peter hit a golf ball from the top of the hill. The height of the ball above the ground is given in the table below:

Time (s)	1	2	3	4	5
Height (m)	52.5	73.2	74.6	55.8	16.1

a) Enter the data in and create a scatter plot. Describe the pattern that the data displays.

Quadratic

b) Determine the equation of the regression function that models the data.

$y = -10.67x^2 + 51.41x + 11$

c) Suggest an appropriate domain and range for this data.

domain $\{x / 0 \leq x \leq 5, x \in R\}$

range $\{y / 6 \leq y \leq 76, y \in R\}$

d) Use your equation to determine the *height* of the ball in

i) 0 seconds

$y = 11$ metres

ii) 2.5 seconds

76.58 m

e) How many seconds did it take the ball to hit the ground?

5.31 secs.

f) When did the ball reach a height of 60.0 metres for the first time?

1.27 secs

g) How long did the ball remain above a height of 60 metres?

$3.84 - 1.27$

$= 2.57$ secs

Working with Polynomial Functions when given the Equation

If you are given the polynomial equation, you already know what type of function it is. Just enter it into the "$y =$" menu and adjust your window settings so that you can see the graph. You can use the same calculator functions as you've used before to answer questions about the polynomial function.

Example 3

A spherical balloon is being inflated. The time it takes to reach a certain volume can be modelled by the equation

y-intercept

$$V = 4.19t^3 + 25.13t^2 + 50.27t + 33.51$$

where V represents the volume of the balloon in cubic centimetres and t represents time in seconds.

a) Determine the volume of the balloon at 2.7 seconds.

$x = 2.7$

$y = 434.91\ cm^3$

b) Determine the volume of the balloon at 10.5 seconds

time : $x = 10.5$ sec.

Volume : $y = 8182.38\ cm^3$

c) How long does it take the balloon to reach a volume of 1000.00 cm^3?

4.20 secs.

d) Explain why the domain and range of the function is limited in this situation.

Negative volume is not possible in this situation.