

## Lesson 5.5 Exercises, pages 397–404

### A

3. Determine whether each ordered pair is a solution of the system of equations.

a)  $y = -x^2 + 10$  ①

$x - y = 2$  ②

(3, 1)

Substitute:  $x = 3, y = 1$  in:

Equation ①: L.S. = 1; R.S. = 1

Equation ②: L.S. = 2, R.S. = 2

The ordered pair is

a solution.

b)  $y = x^2 + 3x - 4$  ①

$y = 2x^2 - 5x + 2$  ②

(2, 6)

Substitute:  $x = 2, y = 6$  in:

Equation ①: L.S. = 6, R.S. = 6

Equation ②: L.S. = 6, R.S. = 0

The ordered pair is not

a solution.

4. Two numbers are related:

The sum of the first number and the square of a second number is 18.

The difference between the square of the second number and twice the first number is 12.

Which system below models this relationship?

a)  $(x + y)^2 = 18$       b)  $x + y^2 = 18$       c)  $x + 2y = 18$

$x^2 - 2y = 12$        $y^2 - 2x = 12$        $2y - x^2 = 18$

The first statement is modelled by  $x + y^2 = 18$ .

The second statement is modelled by  $y^2 - 2x = 12$ .

So, the system in part b is correct.

### B

5. Solve each linear-quadratic system algebraically.

a)  $y = x + 4$  ①

$y = x^2 + x$  ②

From equation ①, substitute  $y = x + 4$  in equation ②.

$x + 4 = x^2 + x$

$x^2 = 4$

So,  $x = -2$  or  $x = 2$

Substitute each value of  $x$  in equation ①.

When  $x = -2$ :

$y = -2 + 4$

$y = 2$

When  $x = 2$ :

$y = 2 + 4$

$y = 6$

The solutions are:  $(-2, 2)$  and  $(2, 6)$

Substitute each solution in each equation to verify.

b)  $y = -x + 5$  ①

$y = (x + 1)^2$  ②

From equation ①, substitute  $y = -x + 5$  in equation ②.

$$-x + 5 = (x + 1)^2$$

$$-x + 5 = x^2 + 2x + 1$$

$$x^2 + 3x - 4 = 0$$

$$(x + 4)(x - 1) = 0$$

So,  $x = -4$  or  $x = 1$

Substitute each value of  $x$  in equation ①.

When  $x = -4$ :

$$y = -(-4) + 5$$

$$y = 9$$

When  $x = 1$ :

$$y = -1 + 5$$

$$y = 4$$

The solutions are:  $(-4, 9)$  and  $(1, 4)$

Substitute each solution in each equation to verify.

c)  $y = 3x - 2$  ①

$y = x^2 + 4x - 2$  ②

From equation ①, substitute  $y = 3x - 2$  in equation ②.

$$3x - 2 = x^2 + 4x - 2$$

$$x^2 + x = 0$$

$$x(x + 1) = 0$$

So,  $x = 0$  or  $x = -1$

Substitute each value of  $x$  in equation ①.

When  $x = 0$ :

$$y = 3(0) - 2$$

$$y = -2$$

When  $x = -1$ :

$$y = 3(-1) - 2$$

$$y = -5$$

The solutions are:  $(0, -2)$  and  $(-1, -5)$

Substitute each solution in each equation to verify.

6. Two numbers are related:

The first number minus 12 is equal to the second number.

The square of the first number minus 30 times the second number is equal to 360.

a) Create a system of equations to represent this relationship.

Let the numbers be represented by  $x$  and  $y$  respectively.

A system is:

$$x - 12 = y$$
 ①

$$x^2 - 30y = 360$$
 ②

b) Solve the system to determine the numbers.

From equation ①, substitute  $y = x - 12$  in equation ②.

$$x^2 - 30(x - 12) = 360$$

$$x^2 - 30x = 0$$

$$x(x - 30) = 0$$

So,  $x = 0$  or  $x = 30$

Substitute each value of  $x$  in equation ①.

When  $x = 0$ :

$$0 - 12 = y$$

$$y = -12$$

When  $x = 30$ :

$$30 - 12 = y$$

$$y = 18$$

The numbers are: 0 and  $-12$ ; or 30 and 18

Substitute each pair of numbers in the problem statements to verify.

7. Solve each quadratic-quadratic system algebraically. Verify each solution using graphing technology.

a)  $y = x^2 + 4$  ①

$$y = -x^2 + 12$$
 ②

From equation ①, substitute  $y = x^2 + 4$  in equation ②.

$$x^2 + 4 = -x^2 + 12$$

$$2x^2 = 8$$

$$x^2 = 4$$

So,  $x = -2$  or  $x = 2$

Substitute each value of  $x$  in equation ①.

When  $x = -2$ :

$$y = (-2)^2 + 4$$

$$y = 8$$

When  $x = 2$ :

$$y = 2^2 + 4$$

$$y = 8$$

The solutions are:  $(-2, 8)$  and  $(2, 8)$

b)  $y = 2(x + 4)^2$  ①

$$y = \frac{1}{2}(x + 1)^2$$
 ②

From equation ①, substitute  $y = 2(x + 4)^2$  in equation ②.

$$2(x + 4)^2 = \frac{1}{2}(x + 1)^2$$

$$4(x + 4)^2 = (x + 1)^2$$

$$4x^2 + 32x + 64 = x^2 + 2x + 1$$

$$3x^2 + 30x + 63 = 0$$

$$x^2 + 10x + 21 = 0$$

$$(x + 3)(x + 7) = 0$$

So,  $x = -3$  or  $x = -7$

Substitute each value of  $x$  in equation ①.

When  $x = -3$ :

$$y = 2(-3 + 4)^2$$

$$y = 2$$

When  $x = -7$ :

$$y = 2(-7 + 4)^2$$

$$y = 18$$

The solutions are:  $(-3, 2)$  and  $(-7, 18)$

$$\text{c) } y = 2x^2 + 12x + 18 \quad \textcircled{1}$$

$$y = -(x + 3)^2 + 12 \quad \textcircled{2}$$

From equation  $\textcircled{1}$ , substitute  $y = 2x^2 + 12x + 18$  in equation  $\textcircled{2}$ .

$$2x^2 + 12x + 18 = -(x + 3)^2 + 12$$

$$2x^2 + 12x + 18 = -x^2 - 6x - 9 + 12$$

$$3x^2 + 18x + 15 = 0$$

$$x^2 + 6x + 5 = 0$$

$$(x + 5)(x + 1) = 0$$

So,  $x = -5$  or  $x = -1$

Substitute each value of  $x$  in equation  $\textcircled{1}$ .

When  $x = -5$ :

$$y = 2(-5)^2 + 12(-5) + 18$$

$$y = 8$$

When  $x = -1$ :

$$y = 2(-1)^2 + 12(-1) + 18$$

$$y = 8$$

The solutions are:  $(-5, 8)$  and  $(-1, 8)$

Substitute each solution in each equation to verify.

8. Solve each linear-quadratic system algebraically. Verify each solution using graphing technology.

$$\text{a) } y = -2x^2 + 1 \quad \textcircled{1}$$

$$4x + 3y = 12 \quad \textcircled{2}$$

From equation  $\textcircled{1}$ , substitute  $y = -2x^2 + 1$  in equation  $\textcircled{2}$ .

$$4x + 3(-2x^2 + 1) = 12$$

$$4x - 6x^2 - 9 = 0$$

$$6x^2 - 4x + 9 = 0$$

Use the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Substitute: } a = 6, b = -4, c = 9$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(6)(9)}}{2(6)}$$

$$x = \frac{4 \pm \sqrt{-200}}{12}$$

Since the discriminant is negative, there are no real solutions.

$$\text{b) } y = x^2 - 3x + 2 \quad \textcircled{1}$$

$$4x - 4y = 7 \quad \textcircled{2}$$

From equation  $\textcircled{1}$ , substitute  $y = x^2 - 3x + 2$  in equation  $\textcircled{2}$ .

$$4x - 4(x^2 - 3x + 2) = 7$$

$$4x - 4x^2 + 12x - 8 = 7$$

$$4x^2 - 16x + 15 = 0$$

$$(2x - 5)(2x - 3) = 0$$

So,  $x = 2.5$  or  $x = 1.5$

Substitute each value of  $x$  in equation  $\textcircled{1}$ .

When  $x = 2.5$ :

$$y = (2.5)^2 - 3(2.5) + 2$$

$$y = 0.75$$

When  $x = 1.5$ :

$$y = (1.5)^2 - 3(1.5) + 2$$

$$y = -0.25$$

The solutions are:  $(2.5, 0.75)$  and  $(1.5, -0.25)$

9. Solve each quadratic-quadratic system algebraically. Use the quadratic formula when necessary.

a)  $y = 2x^2 - 7x + 3$  ①

$y = \frac{2}{3}(x - 1)^2 + 1$  ②

From equation ①, substitute  $y = 2x^2 - 7x + 3$  in equation ②.

$$2x^2 - 7x + 3 = \frac{2}{3}(x - 1)^2 + 1$$

$$6x^2 - 21x + 9 = 2x^2 - 4x + 2 + 3$$

$$4x^2 - 17x + 4 = 0$$

$$(x - 4)(4x - 1) = 0$$

So,  $x = 4$  or  $x = 0.25$

Substitute each value of  $x$  in equation ①.

When  $x = 4$ :

$$y = 2(4)^2 - 7(4) + 3$$

$$y = 7$$

When  $x = 0.25$ :

$$y = 2(0.25)^2 - 7(0.25) + 3$$

$$y = 1.375$$

The solutions are:  $(4, 7)$  and  $(0.25, 1.375)$

b)  $y = x^2 + 8x + 15$  ①

$y = -2x^2 - 16x + 33$  ②

From equation ①, substitute  $y = x^2 + 8x + 15$  in equation ②.

$$x^2 + 8x + 15 = -2x^2 - 16x + 33$$

$$3x^2 + 24x - 18 = 0$$

$$x^2 + 8x - 6 = 0$$

Use the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Substitute: } a = 1, b = 8, c = -6$$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(-6)}}{2(1)}$$

$$x = \frac{-8 \pm \sqrt{88}}{2}$$

$$x = -4 \pm \sqrt{22}$$

Substitute each value of  $x$  in equation ①.

When  $x = -4 + \sqrt{22}$ :

$$y = (-4 + \sqrt{22})^2 + 8(-4 + \sqrt{22}) + 15$$

$$y = 16 - 8\sqrt{22} + 22 - 32 + 8\sqrt{22} + 15$$

$$y = 21$$

When  $x = -4 - \sqrt{22}$ :

$$y = (-4 - \sqrt{22})^2 + 8(-4 - \sqrt{22}) + 15$$

$$y = 16 + 8\sqrt{22} + 22 - 32 - 8\sqrt{22} + 15$$

$$y = 21$$

The solutions are:  $(-4 + \sqrt{22}, 21)$  and  $(-4 - \sqrt{22}, 21)$

$$\text{c) } y = -2(x + 4)^2 - 5 \quad \textcircled{1}$$

$$y = -2x^2 - 16x - 37 \quad \textcircled{2}$$

From equation  $\textcircled{1}$ , substitute  $y = -2(x + 4)^2 - 5$  in equation  $\textcircled{2}$ .

$$-2(x + 4)^2 - 5 = -2x^2 - 16x - 37$$

$$-2x^2 - 16x - 37 = -2x^2 - 16x - 37$$

Since the left side is equal to the right side for all values of  $x$ , there are infinite solutions.

$$\text{d) } y = x^2 + 3x - 2 \quad \textcircled{1}$$

$$y = -x^2 + 4x - 3 \quad \textcircled{2}$$

From equation  $\textcircled{1}$ , substitute  $y = x^2 + 3x - 2$  in equation  $\textcircled{2}$ .

$$x^2 + 3x - 2 = -x^2 + 4x - 3$$

$$2x^2 - x + 1 = 0$$

Use the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Substitute: } a = 2, b = -1, c = 1$$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(2)(1)}}{2(2)}$$

$$x = \frac{1 \pm \sqrt{-7}}{2}$$

Since the discriminant is negative, there are no real solutions.

**10.** Two numbers are related in this way:

The number 1 is subtracted from the first number, the difference is squared, then doubled; the result is equal to the second number.

The number 1 is added to the first number, and the sum is squared; the result is equal to 4 minus the second number.

Determine the numbers. Explain the strategy you used.

Let the numbers be represented by  $x$  and  $y$  respectively.

A system is:

$$2(x - 1)^2 = y \quad \textcircled{1}$$

$$(x + 1)^2 = 4 - y \quad \textcircled{2}$$

From equation  $\textcircled{1}$ , substitute  $y = 2(x - 1)^2$  in equation  $\textcircled{2}$ .

$$(x + 1)^2 = 4 - 2(x - 1)^2$$

$$x^2 + 2x + 1 = 4 - 2x^2 + 4x - 2$$

$$3x^2 - 2x - 1 = 0$$

$$(x - 1)(3x + 1) = 0$$

$$\text{So, } x = 1 \text{ or } x = -\frac{1}{3}$$

Substitute each value of  $x$  in equation  $\textcircled{1}$ .

$$\text{When } x = 1:$$

$$\text{When } x = -\frac{1}{3}:$$

$$2(1 - 1)^2 = y$$

$$2\left(-\frac{1}{3} - 1\right)^2 = y$$

$$y = 0$$

$$y = \frac{32}{9}$$

The numbers are: 1 and 0; or  $-\frac{1}{3}$  and  $\frac{32}{9}$

- 11.** An emergency flare is propelled into the sky from a spot on the ground. The path of the flare is modelled by the equation  $y = -0.096(x - 25)^2 + 60$ , where  $y$  metres is the height of the flare when its horizontal distance from where it was propelled is  $x$  metres. A telescope is placed at the spot from which the flare was propelled. The line of sight from the telescope is modelled by the equation  $8x - 10y = -15$ .

- a) Solve the system formed by the two equations. Give the answers to the nearest tenth of a unit.

$$y = -0.096(x - 25)^2 + 60 \quad \textcircled{1}$$

$$8x - 10y = -15 \quad \textcircled{2}$$

From equation  $\textcircled{1}$ , substitute  $y = -0.096(x - 25)^2 + 60$  in equation  $\textcircled{2}$ .

$$8x - 10(-0.096(x - 25)^2 + 60) = -15$$

$$8x + 0.96x^2 - 48x + 600 - 600 + 15 = 0$$

$$0.96x^2 - 40x + 15 = 0$$

Use the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Substitute: } a = 0.96, b = -40, c = 15$$

$$x = \frac{40 \pm \sqrt{(-40)^2 - 4(0.96)(15)}}{2(0.96)}$$

$$x = \frac{40 \pm \sqrt{1542.4}}{1.92}$$

$$x = 41.2882 \dots \quad \text{or} \quad x = 0.3784 \dots$$

Substitute each value of  $x$  in equation  $\textcircled{2}$ .

When  $x = 41.2882 \dots$

$$8(41.2882 \dots) - 10y = -15$$

$$10y = 345.3058 \dots$$

$$y = 34.5305 \dots$$

When  $x = 0.3784 \dots$

$$8(0.3784 \dots) - 10y = -15$$

$$10y = 18.0274 \dots$$

$$y = 1.8027 \dots$$

The solutions are approximately: (41.3, 34.5) and (0.4, 1.8)

- b) Explain the meaning of the solution of the system.

The flare is seen through the telescope when the flare has travelled approximately 0.4 m horizontally and 1.8 m vertically; and when the flare has travelled approximately 41.3 m horizontally and 34.5 m vertically.

**C**

- 12.** After a football is kicked, it reaches a maximum height of 14 m and it hits the ground 32 m from where it was kicked. After a soccer ball is kicked, it reaches a maximum height of 8 m and it hits the ground 38 m from where it was kicked. The paths of both balls are parabolas.

- a) Create an equation that represents the path of the football.  
Let  $(0, 0)$  represent the initial position of the ball.

Visualize a coordinate grid. Assume the football was on the ground when it was kicked. So, the parabola has  $x$ -intercepts of 0 and 32. The  $x$ -coordinate of its maximum point is 16, so its vertex has coordinates  $(16, 14)$ .

The equation of the parabola has the form:

$$y = ax(x - 32) \quad \text{Substitute: } x = 16, y = 14$$

$$14 = a(16)(16 - 32)$$

$$a = -\frac{7}{128}$$

$$\text{An equation is: } y = -\frac{7}{128}x(x - 32)$$

- b) Create an equation that represents the path of the soccer ball.  
Let  $(0, 0)$  represent the initial position of the ball.

Visualize a coordinate grid. Assume the soccer ball was on the ground when it was kicked. So, the parabola has  $x$ -intercepts of 0 and 38. The  $x$ -coordinate of its maximum point is 19, so its vertex has coordinates  $(19, 8)$ .

The equation of the parabola has the form:

$$y = ax(x - 38) \quad \text{Substitute: } x = 19, y = 8$$

$$8 = a(19)(19 - 38)$$

$$a = -\frac{8}{361}$$

$$\text{An equation is: } y = -\frac{8}{361}x(x - 38)$$

- c) To the nearest tenth of a metre, determine the horizontal distance that both balls have travelled when they reach the same height.

Solve the system formed by the equations in parts a and b.

$$y = -\frac{7}{128}x(x - 32) \quad \textcircled{1}$$

$$y = -\frac{8}{361}x(x - 38) \quad \textcircled{2}$$

From equation  $\textcircled{1}$ , substitute  $y = -\frac{7}{128}x(x - 32)$  in equation  $\textcircled{2}$ .

$$-\frac{7}{128}x(x - 32) = -\frac{8}{361}x(x - 38)$$

$$361(7)x(x - 32) = 128(8)x(x - 38)$$

$$2527x^2 - 80\,864x = 1024x^2 - 38\,912x$$

$$x(1503x - 41\,952) = 0$$

$$x = 0 \quad \text{or} \quad x = 27.9121 \dots$$

So, the balls have travelled approximately 27.9 m horizontally when they reach the same height.